

# Yew Tree Primary School – Maths Calculations Progression and Guidance








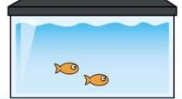
**February 2024**

This calculation progression and guidance reflects the methods taught pictorially, concretely and abstractly to the maths National Curriculum objectives (2014) and the Early Years Framework (2021). Teachers should use this as a resource to help with their planning as well as guiding children to build and develop their mathematical skills and methods.

Due to the Covid-19 crisis, Yew Tree Primary School has implemented recovery lessons for missed learning in the maths curriculum. Teachers should refer to these methods and apply them to the preferred year group the recovery lessons are from. This will help to ensure children are being taught these efficient calculation methods to achieve the National Curriculum maths objectives.

**Addition**

**YEAR N**

<b>Objective and Strategy</b>	<b>Concrete</b>	<b>Pictorial</b>	<b>Abstract</b>
<p>Experiment with own symbols and marks, including numbers</p>	<p>Provide concrete resources to explore number symbols and quantities.</p> 	<p>Play games that introduce children to number symbols.</p> 	<p>Encourage children to record their own symbols and marks and to explain them.</p> 
<p>Solve real world mathematical problems with numbers up to five.</p>	<p>Play games that lend themselves to mathematical problems. 'How many skittles are left standing?'</p> 	<p>Make visual, practical displays showing different ways of making numbers to 6 so that children can investigate these.</p> <p>Have a sustained focus on each number to 6.</p> 	
<p>Compare quantities using language: 'more than'.</p>	<p>Support children to solve problems using fingers and objects. 'There are four of you, but there aren't enough bears'. How many more bears do we need?</p> 	<p>Draw attention to differences and changes in amounts through stories - The enormous Turnip, Mouse Count.</p> <p>Provide resources for children to explore quantities.</p> 	<p>Discuss mathematical ideas.</p> <p>'I think Adam has more crackers...'</p> <p>'I have 3, who has more counters than me?'</p> <p>'Lets add one more fish, now we have two'</p> 

**Addition**

**YEAR R**

Compare quantities using language: 'more than', 'the same as', 'equal'.

Provide a range of collections to compare, starting with a very different number of things. Include 'more' small things and 'fewer' large things, spread them out and bunch them up, to draw attention to the number not the size of things or the space they take up.



Use pictures to compare quantities.



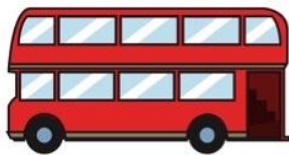
Comparing



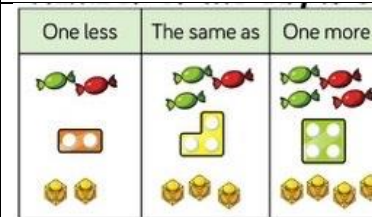
Distribute items evenly, eg: put 3 in each bag, give each child 2 objects. Make deliberate mistakes in order to provoke discussion. Tell a story of a character distributing snacks unfairly and invite a child to make sure everyone has the same.

Understand 'one more than' relationship between consecutive numbers.

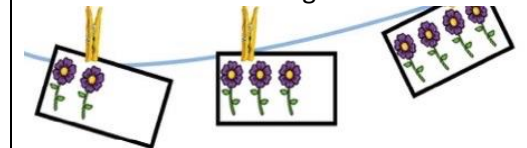
Use concrete resources to add one more.



2 on the bus one more makes 3 on the bus.



Hide a card and for the children to work out which one is missing.



Children to explain choice, for example 3 is missing because it is 1 more than 2.

Explore the composition of numbers to 5 before moving onto larger numbers.

Focus on composition of numbers to 5 before moving onto larger numbers.

Including double facts

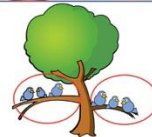
Use cubes, counters, objects of interest to add two sets together as a group.



3 bees and 1 ant. How many altogether?

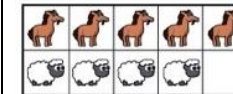


Provide a range of visual models of numbers: for example, six as double three birds on the tree.



Using pictures to add two numbers together as a group.


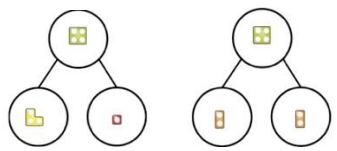
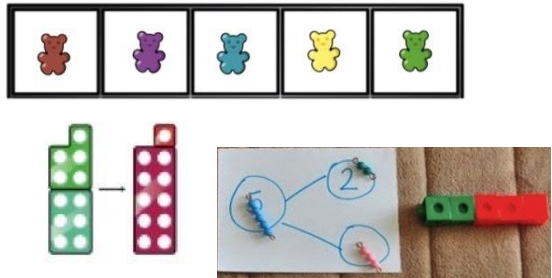
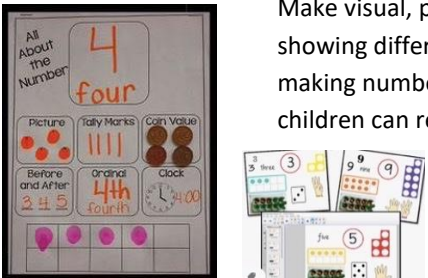

Model conceptual subitising: 'well, there are 3 on this branch and three here, so there must be 6'.

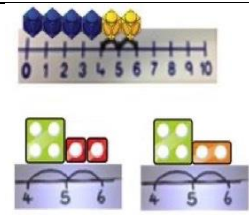



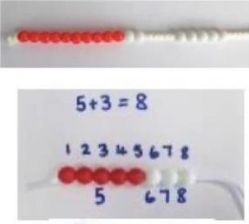
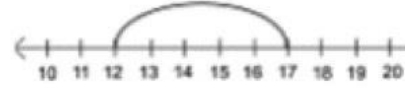

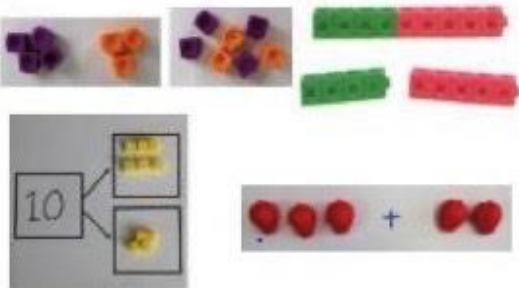
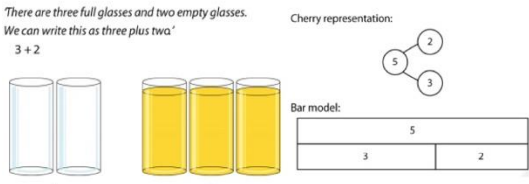


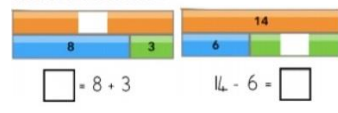
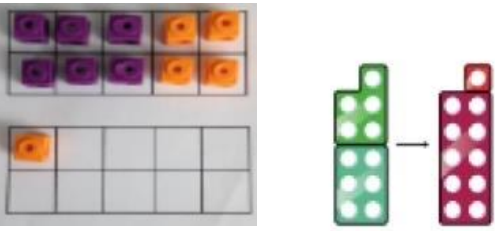
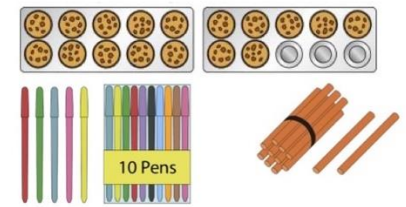
There are  horses.

There are  sheep.

There are  animals altogether.

<p>Combining two parts to make a whole: emphasising the parts.</p>	<p>Use stories and concrete resources to emphasise an understanding of the parts with the whole: 'Well there were eight bananas altogether. Two were eaten and six were not.'</p>  <p>Plan games which involve partitioning and recombining sets. For example, kick 5 footballs, aiming for the goal. How many go in and how many don't?</p>	<p>Provide a range of visual models of part whole to develop understanding - as above.</p>	<p>Develop an understanding of parts adding up to a whole using models such as 5 frames, 10 frames, part whole.</p> 
<p>Automatically recall number bonds for numbers 0-10.</p>	<p>Make amounts using concrete resources. Use models including; part whole, 5 frames, 10 frames, Numicon, dominoes.</p> 	<p>Have a sustained focus on each number to 10.</p> <p>Make visual, practical displays showing different ways of making numbers to 10 so that children can refer to these.</p> 	<p>Children solve problems relating to number bonds for numbers 0-10.</p> 

<b>Addition</b>		<b>YEAR 1</b>	
<p><b>Objective and Strategy</b></p> <p>Counting on adding small amounts with an understanding that counting on from the biggest number is the most</p>	<p><b>Concrete</b></p>  <p>Counting on using number lines and concrete resources.</p>	<p><b>Pictorial</b></p> <p>Use a number line to count on in ones or in one jump to find the answer. Starting at the larger number on the number line.</p> 	<p><b>Abstract</b></p> <p>5+12= Place the larger number in your head and count on the smaller number to find the answer.</p> <p>Counting on using twos. The abstract number line: What is 2 more than 4?</p>

<p>efficient strategy.</p>	 <p>Start with the larger number on the bead string and count on to find the smaller number, 1 by 1 to find the answer.</p>	<p><math>12 + 5 = 17</math></p> 	<p>What is the sum of 2 and 4? What is the total of 4 and 2? <math>4 + 2</math></p> 
<p>Combining two parts to make a whole: part-whole model</p>	<p>Use part part whole model Use cubes to add two numbers together as a group or in a bar.</p> 	<p>Using pictures to add two numbers together as a group or in a bar.</p> <p><i>There are three full glasses and two empty glasses. We can write this as three plus two.</i> <math>3 + 2</math></p>  <p><i>There are two empty glasses and three full glasses. We can write this as two plus three.</i> <math>2 + 3</math></p> 	<p><math>4 + 3 = 7</math> (4 is a part, 3 is a part, 7 is a whole)</p>  <p><math>2 + 3 = 5</math>      <math>3 + 2 = 5</math> <math>5 = 3 + 2</math>      <math>5 = 2 + 3</math> <math>2 + \square = 5</math>      <math>\square + 3 = 5</math> <math>2 + 3 = \square</math></p> <p>Bar models used to show relationship between addition and subtraction.</p>  <p><math>\square = 8 + 3</math>      <math>14 - 6 = \square</math></p>
<p>Regrouping to make ten.</p>	<p>'Make ten' using tens frames and concrete resources. Start with the greater number and then add the smaller number.</p> 	<p>Using pictures of objects and regroup or partition the smaller number to make 10. Starting to apply tens and ones.</p> 	<p>Developing an understanding of grouping ten and applying this into a part part whole model or into equations to develop equality.</p>



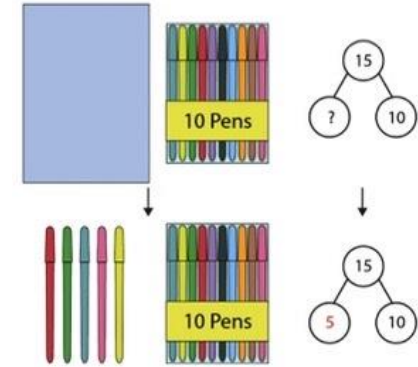
Ten ones are visible:



10s	1s
1	5

'The 1 means one ten, and the 5 means five ones.'

'Sara has fifteen pens. How many are hidden?'



Missing addend equations:

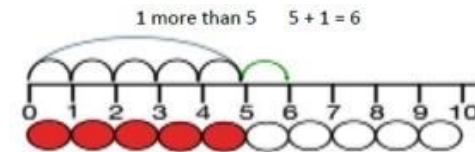
$$\square + 10 = 15 \quad 19 = 10 + \square$$

$$\square + 3 = 13 \quad 11 = 1 + \square$$

Adding 1, 2, 3 more.



Emphasis should be on language: 1 more than...

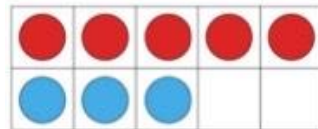


$$\square + 6 = 9 \quad 3 \text{ more than } 6 \text{ is } 9$$

$$6 + 3 = \square \quad 3 \text{ less than } 4, \text{ so } 6 \text{ plus } 3 \text{ is } 1 \text{ less than } 10$$

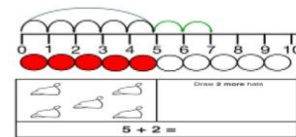
Represent and use number bonds and related subtraction facts within 20.

Teach both addition and subtraction so that pupils begin to identify inverse relationships.



Using tens frames to apply children's knowledge of  $5 + 3 = 8$  and  $3 + 5 = 8$  then applying it to subtraction facts:  $8 - 5 = 3$  and  $8 - 3 = 5$

'There are nine children. Eight of them are reading. How many of them are not reading?'



Using pictorial images and applying them to part, part whole model.

Using number lines and beads.

Emphasis should be on language:

$$8 - 5 = \square \quad 8 \text{ is } 3 \text{ more than } 5$$

$$\square + 6 = 9 \quad 3 \text{ more than } 6 \text{ is } 9$$

$$6 + 3 = \square \quad 3 \text{ less than } 4, \text{ so } 6 \text{ plus } 3 \text{ is } 1 \text{ less than } 10$$

**Addition**

**YEAR 2**

Counting on in multiples of 10.

Using dienes and bead strings to represent multiples of 10.

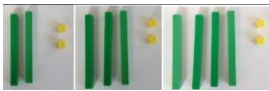


$50=30+20$



Using Numicon to represent 10.

$60=10+10+10+10+10+10$



Counting on with 10s and 1s

Use pictorial representations for base 10.



$3 \text{ tens} + 5 \text{ tens} = \text{---} \text{ tens}$

$30 + 50 = \text{---}$

$20 + 30 = 50$

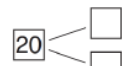
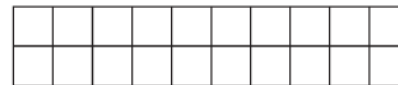
$70 = 50 + 20$

$40 + \square = 60$

Using known number facts to 20.

Part, part, whole

Children explore ways of making numbers with 20.



$\square + \square = 20$

$20 - \square = \square$

$\square + \square = 20$

$20 - \square = \square$

$20 = 17 + 3$

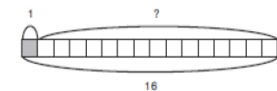
$20 = 3 + 17$

$20 - 3 = 17$

$20 - 17 = 3$

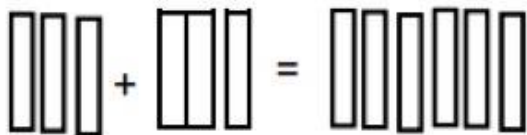
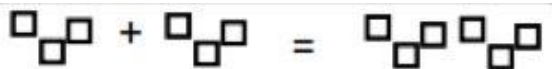
$\square + 1 = 16$        $16 - 1 = \square$

$1 + \square = 16$        $16 - \square = 1$

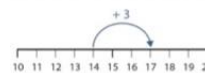


Using known number facts to understand derived facts - up to 100.

Children explore ways of using known facts.



$4 + 3 = 7$



$14 + 3 = 17$

$3 + 4 = 7$

leads to

$30 + 40 = 70$

leads to

$300 + 400 = 700$

Children draw representations of H, T, and O.

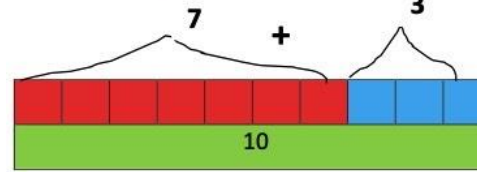
Bar Model



$$3 + 4 = 7$$



$$7 + 3 = 10$$



27	
15	?

12	15
?	

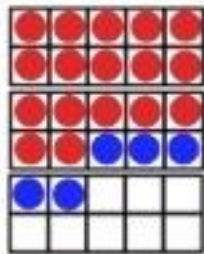
?	
15	?

$$15 + 12 = 27$$

Adding a two-digit number and ones

Make 10 strategy

Continue to develop an understand partitioning and place value.



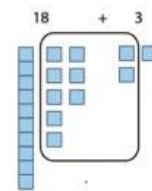
$$17 + 5 = 22$$

Use a 10 frame to make a 'magic 10'.

Children explore the pattern.

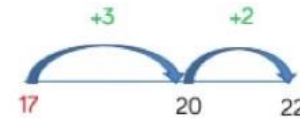
$$17 + 5 = 22$$

$$27 + 5 = 32$$

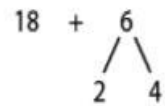


$$18 + 3 = 18 + 2 + 1 = 20 + 1 = 21$$

Using dienes, Part Whole and number lines to model.



$$18 + 6 = \square$$

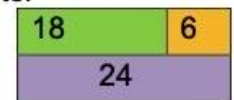


Explore related facts:

$$6 + \square = 24$$

$$24 - 6 = \square$$

$$\square - 18 = 6$$



Adding a two-digit number and a tens number.



$17 + 10 = 27$

Explore that the ones digit does not change.

Using Dienes pictures

We had 3 tens and 2 ones. Ten more gives us 4 tens and 2 ones.

Use number squares

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

'Ten more than forty-two is fifty-two'

28 38 48

$23 + 10 = \square$

$33 + \square = 43$

$\square + 10 = 53$

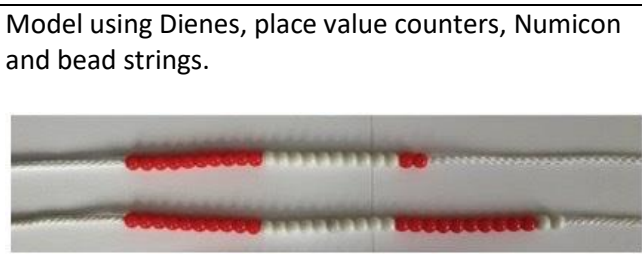
27 + 10 = 37

27 + 20 = 47

27 +  $\square$  = 57

Adding two two-digit numbers.

Partitioning a number into 10s and 1s.



Use number lines to bridge 10. Use part, part, whole where necessary.

$22 + 17 = 39$

Partitioning into 10s and 1s.

$45 + 23$

$40 + 20 = 60$

$5 + 3 = 8$

$60 + 8 = 68$

Adding two two-digit numbers.

Round and adjust (compensation strategy)



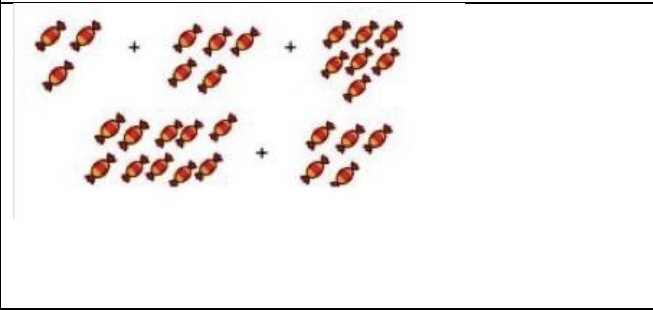
Adding 20 and subtracting 3 is easier than trying to add 17. This is addition by compensation.

$22 + 17 = 39$

Adding 3 single-digit numbers.

$4 + 7 + 6 = 17$

Put 4 and 6 together to make 10. Add on 7.



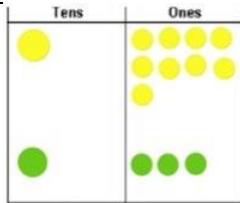
Combine the 6 + 4 to make 10, then adding the 7.



Combine to make 10 first if possible, or bridge 10 then add third digit

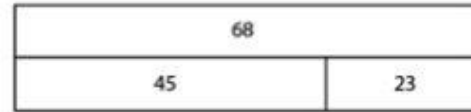
Add 3 groups together. Draw a picture to recombine the groups to make 10.

Introducing column addition - no grouping.



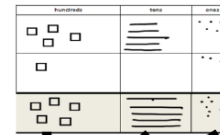
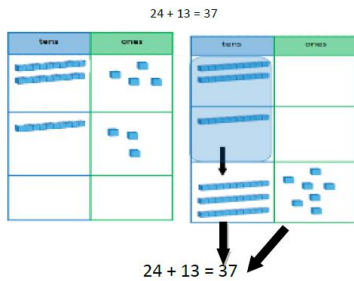
Use place value counters and Dienes to provide support.

Use the bar model to represent the calculation.



Introduce expanded method without exchanging.

$$\begin{array}{r} 18 \\ + 11 \\ \hline 29 \end{array}$$



$$45 + 103 = 558$$

Introducing column addition - grouping.



$$\begin{array}{r} 45 \\ 40 \quad 5 \\ + 23 \\ 20 \quad 3 \\ \hline 60 \quad 8 \\ 60 + 8 = 68 \end{array}$$

First partition into 10s and 1s, regroup where required. Add the 10s. Add the 1s. Then add the 10s and 1s.

hundreds	tens	ones
3	5	8
	3	7
3	9	5



Introduce expanded method by regrouping without exchanging. Then move onto exchanging with 10s and 1s.

$$\begin{array}{r} 19 \\ + 13 \\ \hline 20 \\ 32 \end{array} \qquad \begin{array}{r} 19 \\ + 13 \\ \hline 32 \\ 1 \end{array}$$

Add numbers mentally, including:

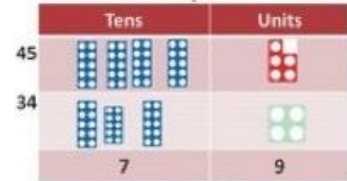
a three-digit number and ones;

a three-digit number and tens;

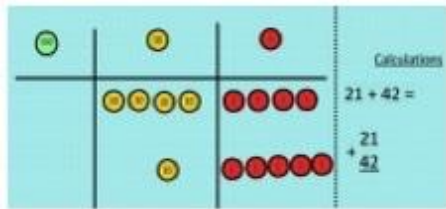
a three-digit number and hundreds



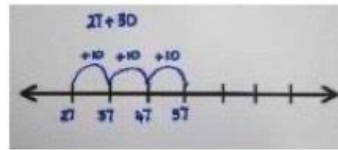
Model using Dienes or Numicon.



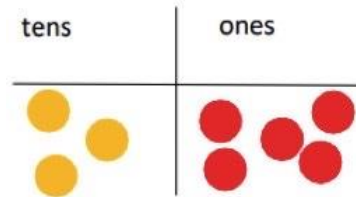
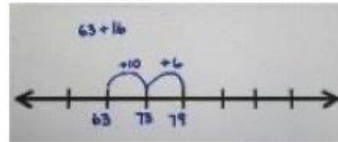
Add together the ones first then the tens.



Move to using place value counters.



Using number lines.



Pupils move to drawing counters using a frame - H T O

$$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$$

No regrouping

345 + 30

274 - 50

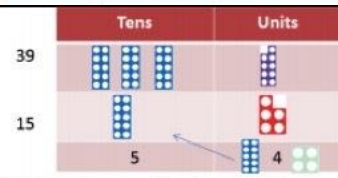
1128 + 300

1312 - 300

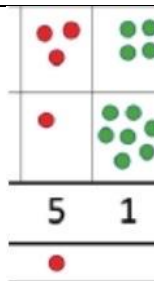
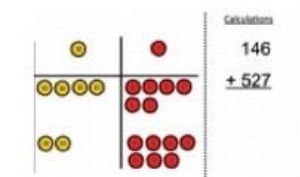
326 + 342

856 - 724

Column addition for calculations up to 4 digits that require regrouping



Exchange ten ones for a ten. Model using Numicon and place value counters.



To further support children's understanding they can draw a representation of the grid. Carrying the one underneath the line.

2	7	5
+3	8	6

With some regrouping

416 + 25

232 - 5

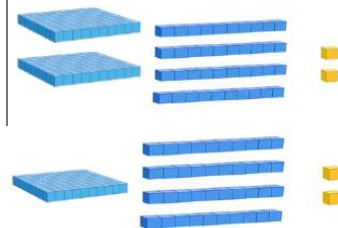
383 + 130

455 - 216

611 + 194

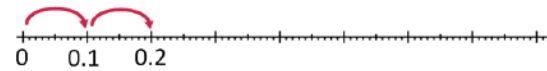
130 - 40

Find 10 or 100 more or less than a given number



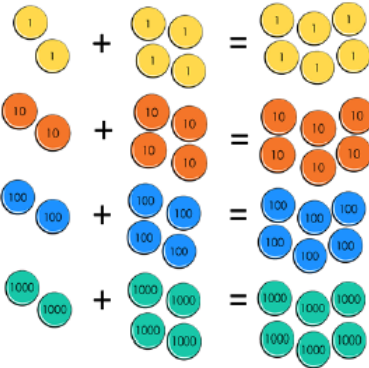
142 + 100 = 242

Count forwards or backwards in steps of 10, 100, and 1000.



E.g.  $990 + 10$  or  $19.9 + 0.1$

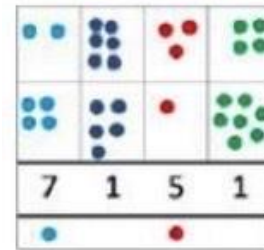
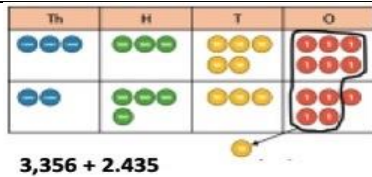
Using known facts and knowledge of place value to derive facts.



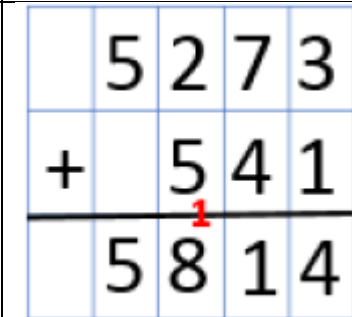
Add and subtract multiples of 10, 100 and 1000 mentally.

$2 + 4 = 6$   
 $20 + 40 = 60$   
 $200 + 400 = 600$   
 $2000 + 4000 = 6000$

Written column methods for addition

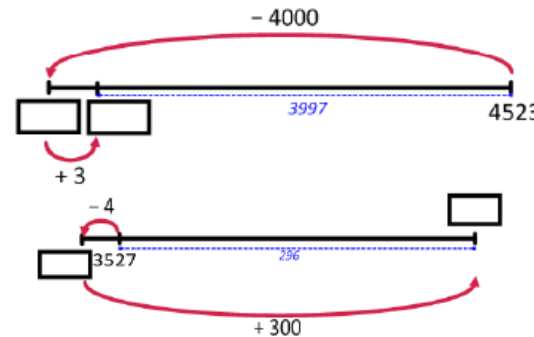


To further support children's understanding they can draw a representation of the grid. Carrying the one underneath the line.



Round and adjust

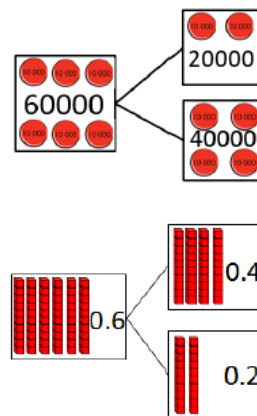
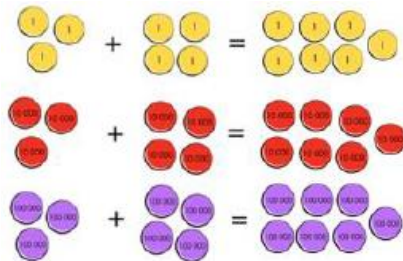
Children continue to use Dienes or place value counters to add, exchanging 10 ones for a ten, 10 tens for a hundred and so on.



$3527 + 296 = 3827 - 4$   
 $3527 + 296 = 3523 + 300$   
 $4523 - 3997 = 523 + 3$



Using known facts and understanding of place value to derive.



$$3 + 4 = 7$$

$$30\ 000 + 40\ 000 = 70\ 000$$

$$300\ 000 + 400\ 000 = 700\ 000$$

$$20\ 000 + 40\ 000 = 60\ 000$$

$$40\ 000 + 20\ 000 = 60\ 000$$

$$60\ 000 - 40\ 000 = 20\ 000$$

$$60\ 000 - 20\ 000 = 40\ 000$$

$$0.6 = 0.2 + 0.4$$

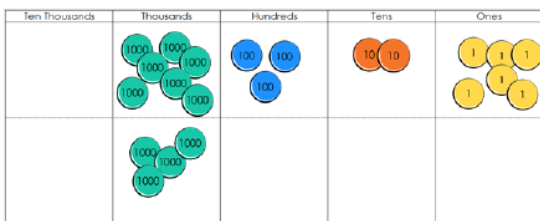
$$0.6 = 0.4 + 0.2$$

$$0.2 = 0.6 - 0.4$$

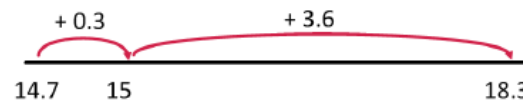
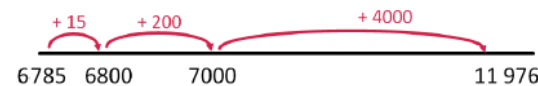
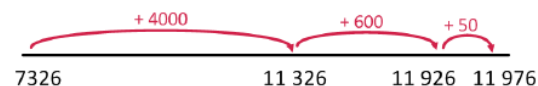
$$0.4 = 0.6 - 0.2$$

Partitioning one number and applying known facts to add.

Using canonical and non-canonical methods.



With place value counters, represent the larger number and then add each place value part of the other number. The image above shows the thousands being added.



$$4650 + 7326 = 7326 + 4000 + 600 + 50$$

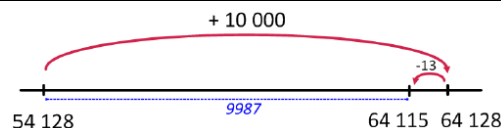
Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count on to a multiple of 10 e.g.

$$6785 + 2325 = 6785 + 15 + 200 + 2110$$

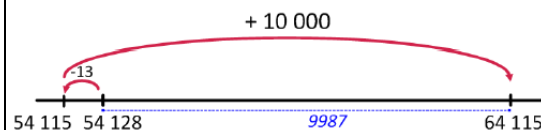
The strategy can be used with decimal numbers, Make one:

$$14.7 + 3.6 = 14.7 + 0.3 + 3.3 = 15 + 3.3$$

Round and adjust using compensation



Pupils should realise that they can adjust first:



$$54\ 128 + 9987 = 54\ 128 + 10\ 000 - 13$$

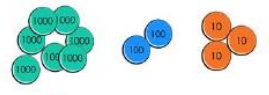
$$= 64\ 128 - 13$$

$$54\ 128 + 9987 = 54\ 128 - 13 + 10\ 000$$

$$= 54\ 115 + 10\ 000$$



Near doubles

$200 + 300 = 500$



Pupils should be aware that the parts can be added in any order.

$7000 + 5000 = 12000$        $30 + 10 = 40$

$160 + 170 = \text{double } 150 + 10 + 20$

$160 + 170 = \text{double } 160 + 10$   
or  $160 + 170 = \text{double } 170 - 10$


$2.5 + 2.6 = \text{double } 2.5 + 0.1$

Partition both numbers and combine the parts

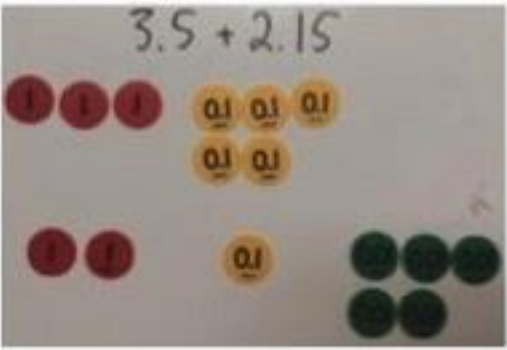
For this method start with the digit of least value because if regrouping happens it will affect the digits of greater value.

Ten Thousands	Thousands	Hundreds	Tens	Ones
3	5	0	0	0
2	1	5	0	0

Combine the counters in each column and regroup as needed:



Ten Thousands	Thousands	Hundreds	Tens	Ones
5	6	5	0	0



	3	4	6	2	3
+		5	5	4	1
<hr/>					
<hr/>					

Add several numbers of increasing complexity.

Including adding money, measures and decimals with different numbers of decimal points.

Tens	Ones	Tenths	Hundredths	Thousandths

$$\begin{array}{r}
 81,059 \\
 3668 \\
 15,301 \\
 + 20,551 \\
 \hline
 120,579
 \end{array}$$

Insert zeros for place holders

$$\begin{array}{r}
 23.361 \\
 9.080 \\
 59.770 \\
 + 1.300 \\
 \hline
 93.511
 \end{array}$$

**Subtraction**

**Year N**

Experiment with own symbols and marks, including numbers

Provide concrete resources to explore number symbols and quantities.



Play games that introduce children to number symbols.



Encourage children to record their own symbols and marks and to explain them.



Solve real world mathematical problems with numbers up to five.

Play games that lend themselves to mathematical problems. 'How many skittles have you managed to knock down?'






Make visual, practical displays showing different ways of making numbers to 5 so that children can investigate these.






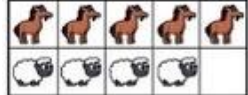
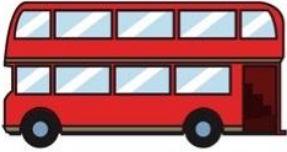






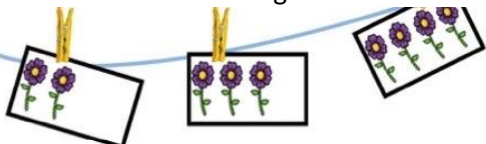



Recite numbers past 5.


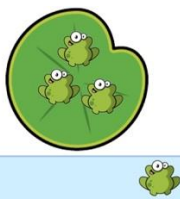
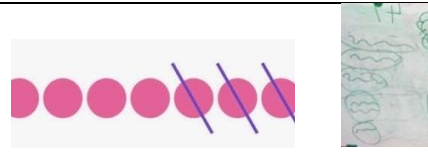
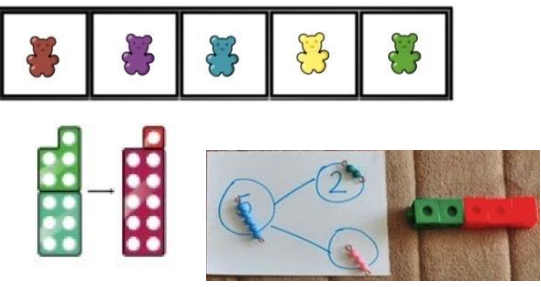
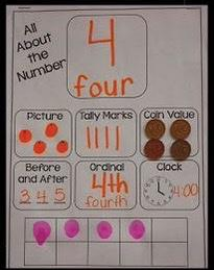


Have a sustained focus on each number to 5.

<p>Compare quantities using language: 'fewer than'.</p>	<p>Support children to solve problems using fingers and objects. 'There are four of you, do we have more of fewer bears.'</p> 	<p>Draw attention to differences and changes in amounts through stories - The enormous Turnip, Mouse Count.</p>  <p>Provide resources for children to explore quantities.</p>	<p>Discuss mathematical ideas. 'I think Adam has fewer crackers...'</p> <p>'I have 3, who has fewer counters than me?'</p> <p>'Lets take a fish, now we have fewer, we have two'</p> 
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**Subtraction**

**Year R**

<p>Compare quantities using language: 'fewer than', 'the same as', 'equal'.</p>	<p>Provide a range of collections to compare, starting with a very different number of things. Include 'more' small things and 'fewer' large things, spread them out and bunch them up, to draw attention to the number not the size of things or the space they take up.</p> 	<p>Use pictures to make direct comparison of quantity.</p> <p>Story Mat</p>  <p>Comparing</p> 	 <p>There are <input type="text"/> horses.</p> <p>There are <input type="text"/> sheep.</p> <p>Are there fewer horses or fewer sheep?</p>			
<p>Understand 'one fewer than' relationship between consecutive numbers.</p>	<p>Use concrete resources to take one away.</p>  <p>3 on the bus one gets off, leaving 2 on the bus.</p>	<table border="1"> <tr> <td data-bbox="945 1141 1034 1300"> <p>One less</p>  </td> <td data-bbox="1034 1141 1124 1300"> <p>The same as</p>  </td> <td data-bbox="1124 1141 1214 1300"> <p>One more</p>  </td> </tr> </table> <p>Focus on language of one fewer.</p>	<p>One less</p> 	<p>The same as</p> 	<p>One more</p> 	<p>Hide a card and for the children to work out which one is missing.</p>  <p>Children to explain choice, for example 3 is missing because it is 1 fewer than 4.</p>
<p>One less</p> 	<p>The same as</p> 	<p>One more</p> 				

<p>Explore the composition of numbers to 10.</p>	<p>Focus on composition of numbers to 5 before moving onto larger numbers.</p> <p>Use cubes, counters and objects of interest to subtract from a set.</p>  <p>5 cherry buns, 3 get eaten, how many are left?</p>	<p>Provide a range of visual models and stories for subtraction.</p>  <p>Using pictures to illustrate subtraction for example, 4 frogs on the lily pad, one jumps off.</p> <p>Encourage children to subitise small amounts.</p>	 <p>Encourage children to record in their own way and explain their jottings.</p>
<p>Automatically recall subtraction facts for numbers 0-10.</p>	<p>Make amounts using concrete resources. Use models including; part whole, 5 frames, 10 frames, Numicon, dominoes.</p> 	<p>Have a sustained focus on each number to 10.</p> <p>Make visual, practical displays showing different ways of making numbers to 10 so that children can refer to these.</p>  	<p>Children solve problems relating to subtraction facts for numbers 0-10.</p> 

**Subtraction**

**YEAR 1**

Year 1  
Taking away  
from the ones.

First      Then      Now

8      -1      7

8 - 1 = 7

4 - 3 = 1

5      -1      4

5 - 1 = 4

Use physical objects, counters, cubes etc to show how objects can be taken away.

6 - 4 = 2

4 - 2 = 2

Children draw the concrete resources they are using and cross out the correct amount.

7 - 3 = 4

37 - 3 = 30

First there were 7 bees on the hive.

Then 3 flew away.

Now there are 4 bees on the hive.

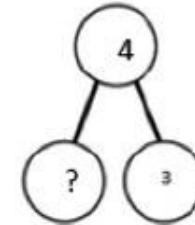
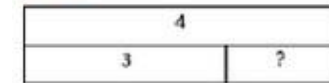


7 - 1 =       2 - 1 =

6 - 1 =       1 - 1 =

4 - 3 =

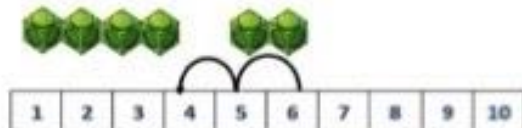
= 4 - 3



Year 1  
Counting back

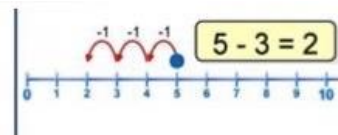
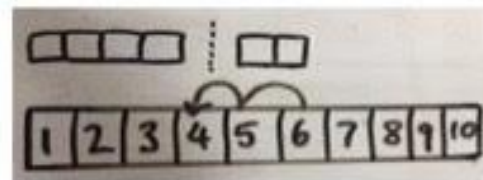
Using number lines or tracks. Make the large number. Move the beads along the bead string as you count backwards in ones.

6 - 2 = 4



13 - 4 =

Count back in ones using a number line to represent what they see pictorially.

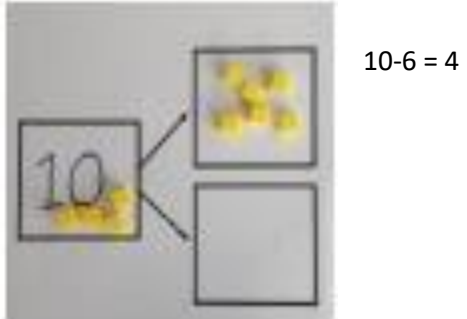


Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

Represent and use subtraction facts within 20.

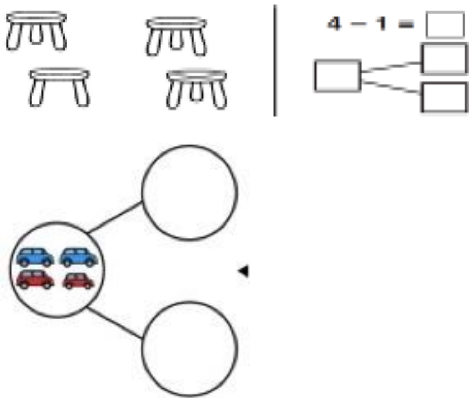
Part-Part Whole model.

Link to addition – use the part whole model to help explain the inverse between addition and subtraction.



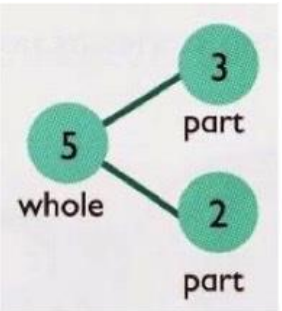
$10 - 6 = 4$

Use a pictorial representation of objects to show the part – part – whole model. Any objects can be used for this.



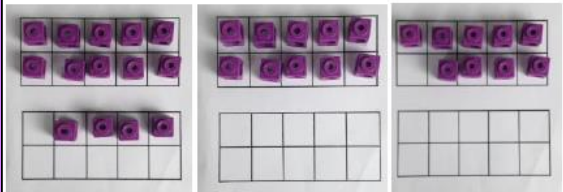
$4 - 1 = \square$

Move to using numbers within the part-part-whole model.



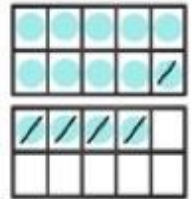
Make Ten Strategy

$14 - 5 =$   
Make 14 on the ten frame. Take 4 away to make ten, then take 1 more away to make 9.

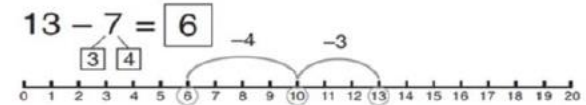


$14 - 5 = 9$

Children represent the ten frame pictorially and discuss what they did to make 10.

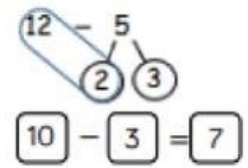


Use number lines. Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken 7 altogether. You have reached your answer.



$13 - 7 = 6$

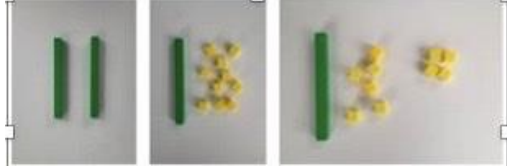
Children to demonstrate how they can make 10 by partitioning the subtrahend.



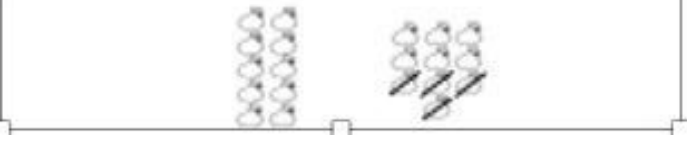
$10 - 3 = 7$

$16 - 7 =$   
How many do we take off first to get to 10?  
How many left to take off?

Regroup ten into ten ones.



Use a place value chart to show how to change a ten into ten ones, use the term 'take and make'.

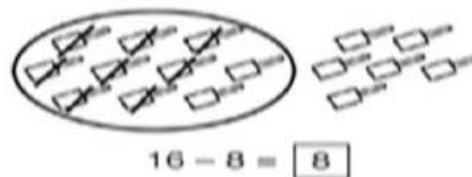
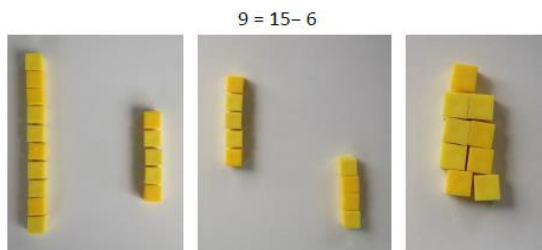


$20 - 4 =$

$20 - 4 = 16$

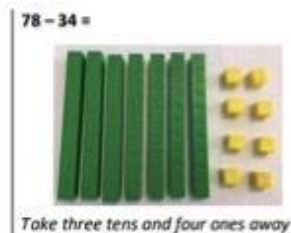
Subtracting from the tens

Subtract from the ten where the one is too small, then combine the remainder with the ones.



Partitioning to subtract without regrouping.

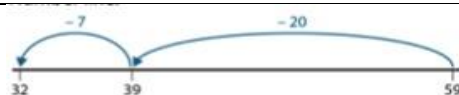
Use Dienes to show how to partition the number when subtracting without regrouping.



$34 - 13 = 21$



$34 - 13 = 21$

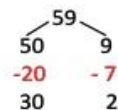


$59 - 27$   
Using number lines and part-part whole model

Part-part-whole diagram:



$59 - 27 = \underline{\quad}$



Partition the number 59  
Partition 27 and subtract the ones and the tens.  
Place the partitioned number back together.

Children draw representations of Dienes and cross off.

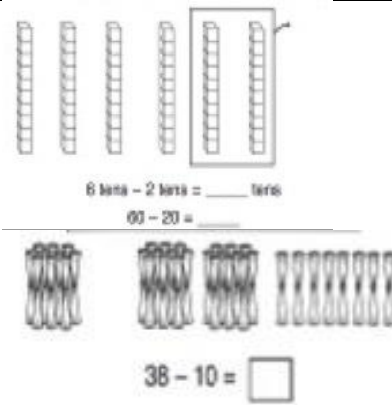
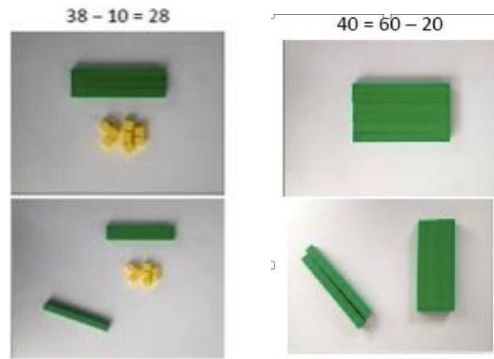


$43 - 21 = 22$

$59 - 27 = 32$

Year 1  
Subtracting  
multiples of  
ten

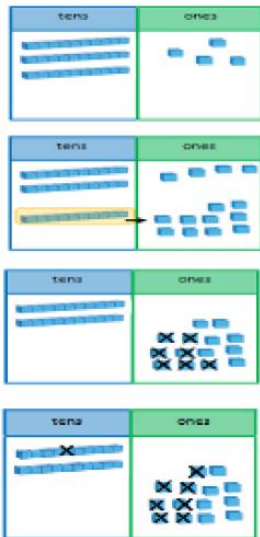
Use Dienes to model subtracting multiples of ten.



$$70 - \square = 30$$

$$43 - \square = 13$$

Year 1  
Column  
method with  
regrouping



$$34 - 17 = 17$$

Children draw the concrete resources they are using and  
cross out the correct amount.

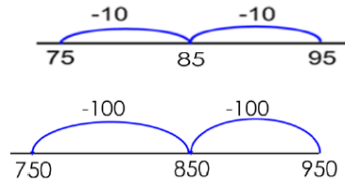
$$46 - 18 =$$

$$24 - 6 =$$

**Subtraction**

**Year 2**

Counting back in multiples of ten and one hundred

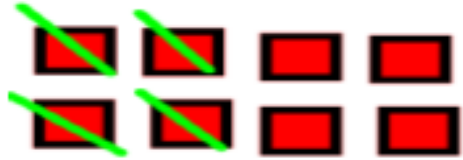


Counting back in multiples of ten and one hundred.

$95 - 20 =$

$950 - 200 =$

Using known fact to explore derived facts



$8 - 4 = 4$

leads to

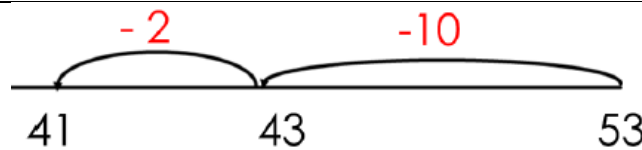
$80 - 40 = 40$

leads to

$800 - 400 = 400$

Subtracting tens and ones by partitioning first.

Use concrete resources to model subtraction by partitioning the number first.

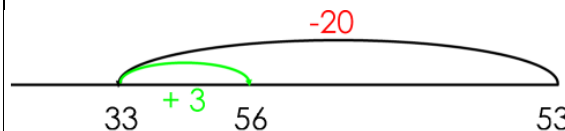
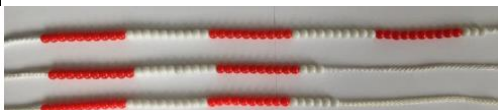


Use number lines to illustrate the partition method.

$53 - 12 = 41$

Round and adjust (compensation strategy)

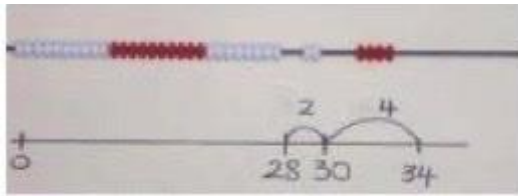
Use concrete resources to model rounding up to the nearest multiple of ten, subtracting and adjusting.



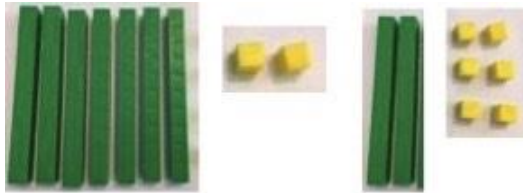
$53 - 17 = 36$

Make ten strategy

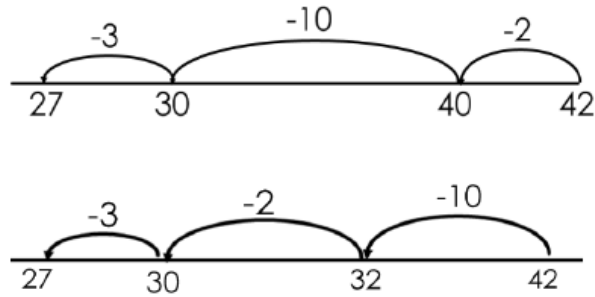
Use concrete resources to model counting to the next ten and the rest.



$$34 - 28 =$$



$$72 - 26 =$$



Use number lines to count to the next ten and the rest.

$$42 - 15 =$$

Subtraction

Year 3

Beginning to use the column method to subtraction without grouping

Use base 10 or Numicon to model.

Use place value counters model how to partition numbers to subtract.

T	U	T	U
30	5	30	5
10	4	10	4
20	1	20	1

Represent methods pictorially.

hundreds	tens	ones
<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>

Calculations

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

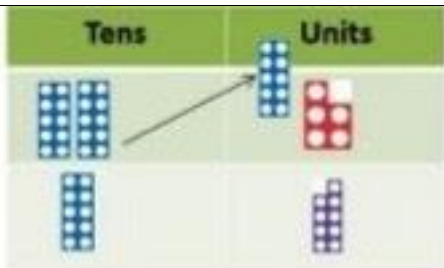
$$47 - 24 = 23$$

$$\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

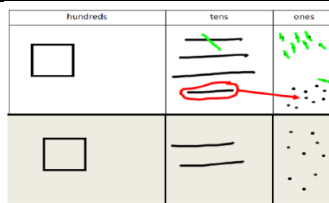
Intermediate step may be needed to lead to clear subtraction understanding.

$$\begin{array}{r} 32 \\ - 12 \\ \hline 20 \end{array}$$

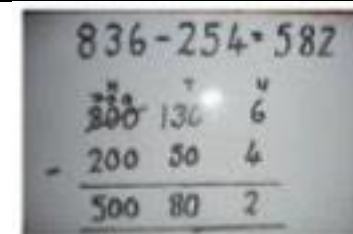
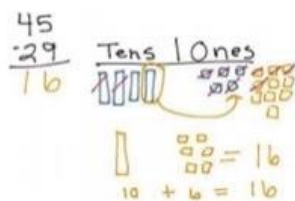
Beginning to use the column method to subtraction with grouping



Begin with base 10 or Numicon. Then move to place value counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.



Children draw base 10 or place value counters and cross off.



Partition into place value columns if needed before moving onto formal methods.

hundreds	tens	ones
8	3	6
-	2	5
5	8	2

## Subtraction

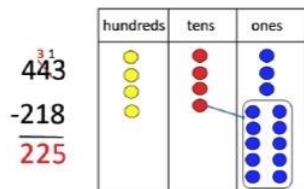
Year 4

Subtracting tens and ones

Subtract with up to 4 digits.

Introduce decimal subtraction through context of money.

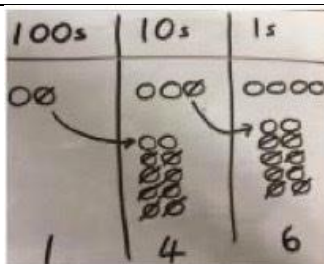
Model the process of exchanging using Numicon, base 10 and place value counters.



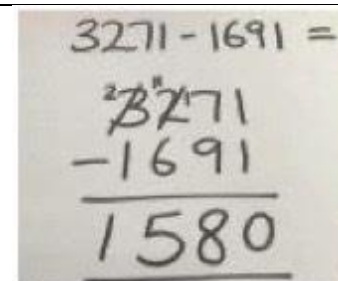
$$\begin{array}{r} 443 \\ -218 \\ \hline 225 \end{array}$$

$$443 - 218 = 225$$

$$4352 - 3271 =$$

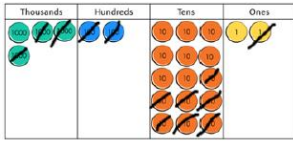
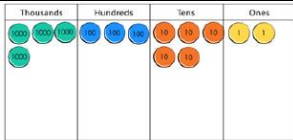


Draw the counters onto a place value grid and show what you have subtracted by crossing out the counters, as well as clearly showing the exchanges you make.



Use the phrase 'take and make' for the exchange.

$$\begin{array}{r} 3271 \\ -1691 \\ \hline 1580 \end{array}$$



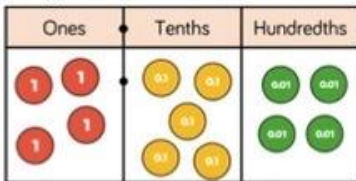
**Subtraction**

**Year 5 & 6**

Subtract with at least 4 digits, including money and measures.

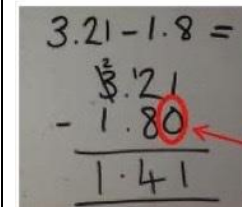
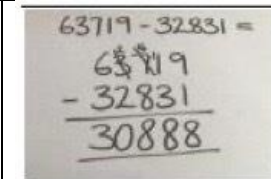
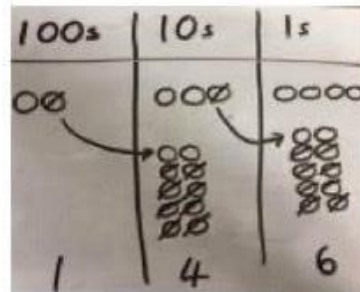


Follow on from year 4 using Numicon, base 10 and place value counters.



4.54 - 1.4 =

Continue to draw the counters onto a place value grid and show what you have subtracted by crossing out the counters, as well as clearly showing the exchanges you make.



Demonstrate the 0 as a place holder.

Subtract with increasingly large and more complex numbers and decimal values

Continue to use place value counters, Numicon, base ten where appropriate.

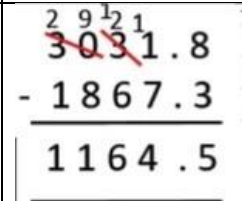
You can use the terms 'exchange' with subtraction but it needs careful consideration.

You can regroup 62 as 50 and 12 (5 tens and 12 ones) instead of 60 and 2 (6 tens and 12 ones). Or you can 'exchange' one of the tens for 10 ones resulting in 5 tens and 12 ones.

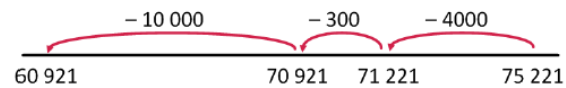
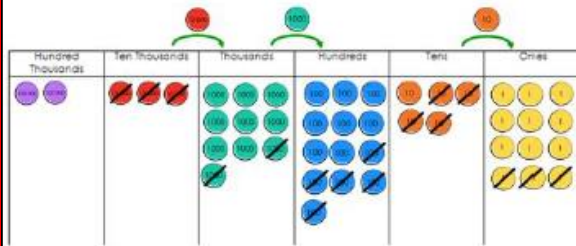
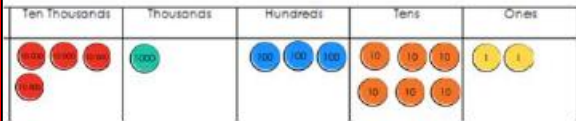
Represent pictorially with a number line, starting on the right and having the arrows jump to the left:



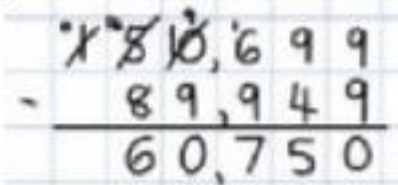
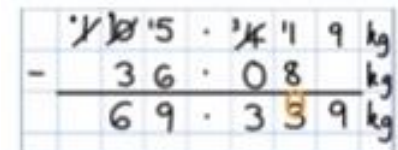
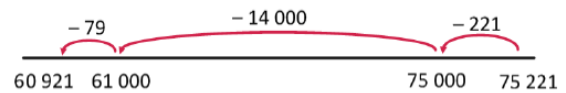
Develop understanding that the parts can be subtracted in any order and the result will be the same:



If you have exchanged, then the number has been regrouped.

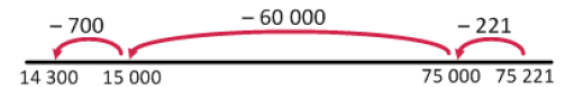


Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count back to a multiple of 10.

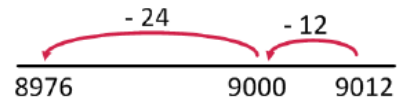


Calculate difference by 'counting back' And 'counting on'

Place the numbers either end of a numberline and work out the difference between them. Select efficient jumps.



Finding the difference is efficient when the numbers are close to each other:



$$75\,221 - 14\,300 =$$

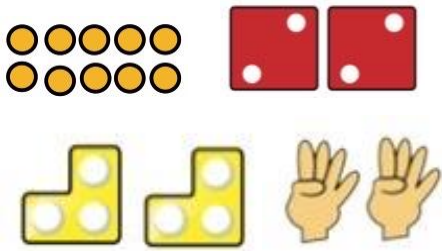
$$9012 - 8976 =$$

## Multiplication

Year R

Doubling using numbers to 5.

Using a range of practical activities and equipment to demonstrate doubling.

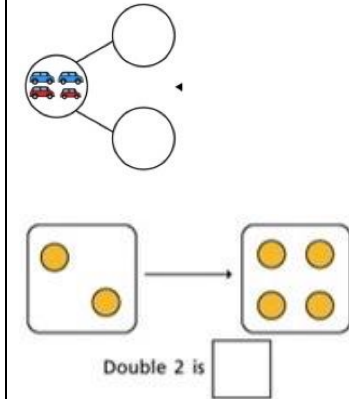


Draw pictures to demonstrate the doubling of numbers.



Encourage children to make their own jottings to explain their understanding.

Use part, part whole to partition and recombine doubles.



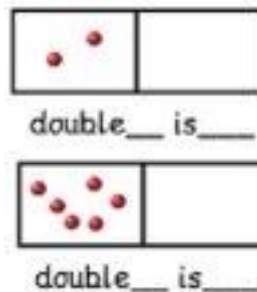
## Multiplication

Year 1

Doubling

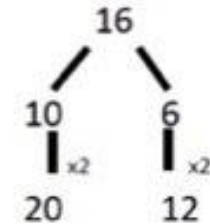
Using a range of practical activities and equipment to demonstrate doubling.

Draw pictures to demonstrate the doubling of numbers.

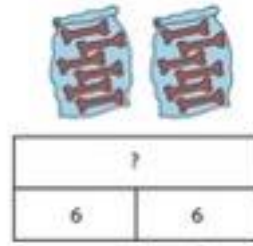
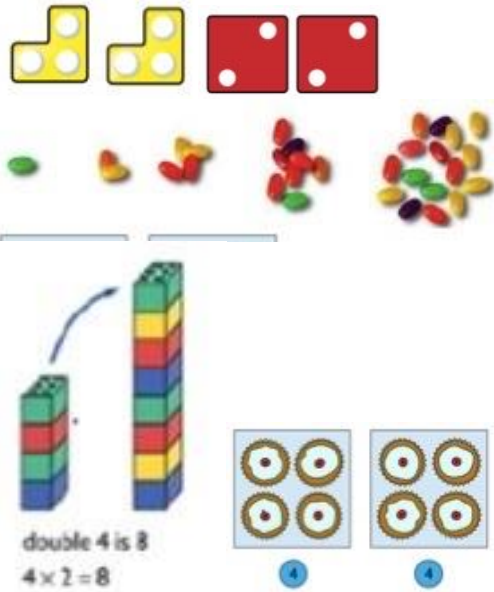


Bar models are useful for demonstrating doubles.

Partition a number and then double each part before recombining it back together.

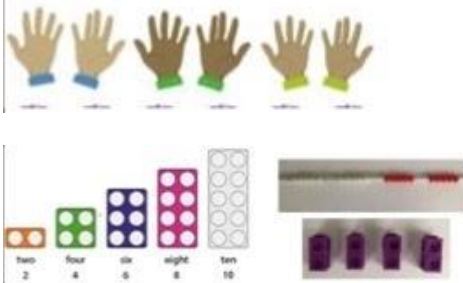


Double 16 =  
 $16 + 16 =$   
 + 16 = 32

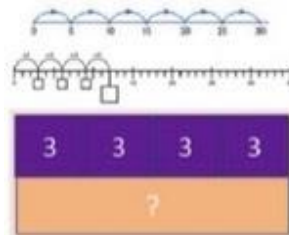


Skip counting in multiples of 2, 5, 10 from zero

Count the groups as children are skip counting.



Visual representations of objects can be demonstrated to show counting in multiples.



Number lines, counting sticks and bar models can also be used to show representations of multiples.

Count in multiples of a number aloud.

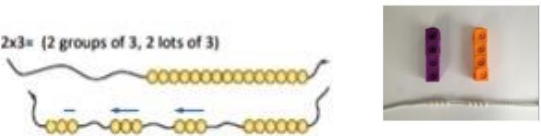
Write sequences of numbers.

2, 4, 6, 8, 10  
5, 10, 15, 20, 25, 30

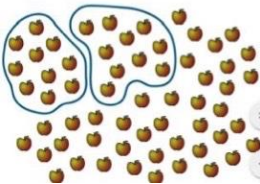
$2 \times 3 = \square$

Making equal groups and counting the total

Children need lots of practice in creating equal groups with practical resources.




Children split objects into different groups.



How many groups of 10 are there?  
What is the total amount?  
 $6 \times 10 =$

Complete the sentences

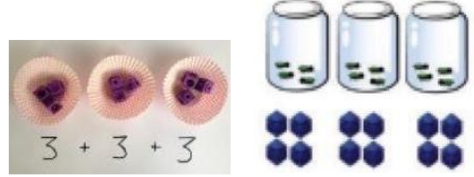

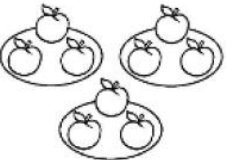


There are \_\_\_ groups of \_\_\_ pencils.

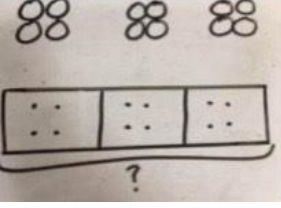
$2 \times 6 = 12$

Solve multiplications using repeated addition


Use different objects to add equal groups.

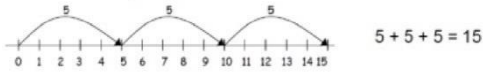
How many apples are there altogether?



There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?




2 + 2 + 2 equals 6



$5 + 5 + 5 = 15$

Use pictorial representations including number lines and bar models.


Write addition sentences to describe objects and pictures.



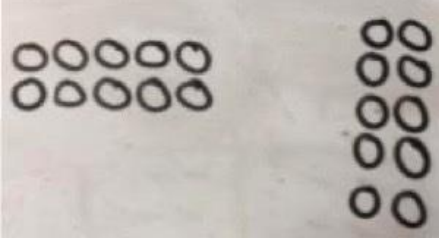
$2 + 2 + 2 + 2 = 8$

Understanding arrays

Use and create arrays to show multiplication sentences.




2 lots of 5      5 lots of 2



Children to draw the arrays pictorially.

Using arrays to write multiplication sentences.



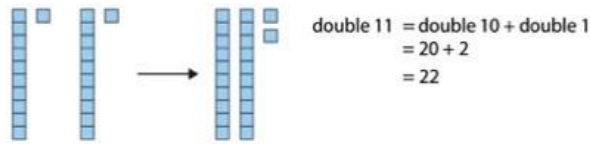
$5 + 5 = 10$   
 $2 + 2 + 2 + 2 + 2 = 10$   
 $2 \times 5 = 10$   
 $5 \times 2 = 10$

**Multiplication**

**Year 2**

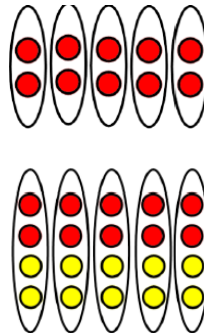
**Doubling**

Model doubling using Dienes and place value counters.



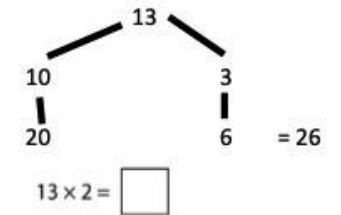
$$\begin{aligned} \text{double } 11 &= \text{double } 10 + \text{double } 1 \\ &= 20 + 2 \\ &= 22 \end{aligned}$$

Draw pictures and representations to show how to double numbers.



Partition a number and then double each part before recombining it back together.

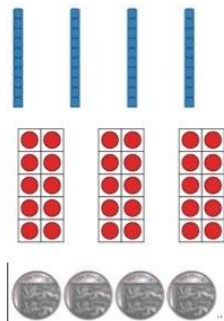
*There are thirteen pairs of socks. How many socks are there altogether?*



**Counting in multiples of 2, 3, 4, 5, 10 from 0.**

Count the groups as children are counting, children may use their fingers as they are skip counting.

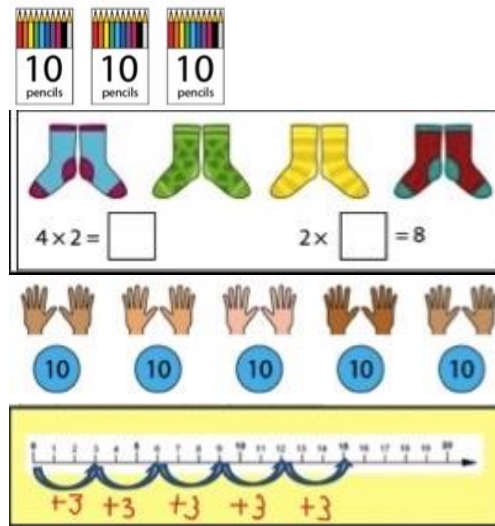
Use a range of resources to demonstrate this.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Number lines, counting sticks and bar modes to show representation of counting in multiples.



Counting in multiples of a number aloud.

*'Fill in the missing numbers.'*

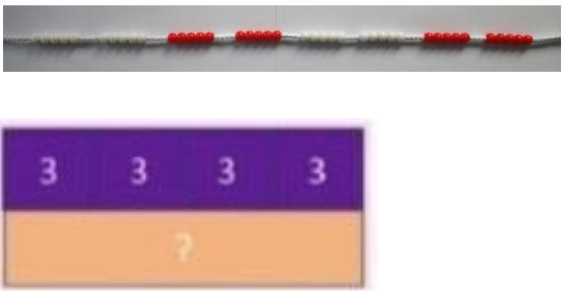
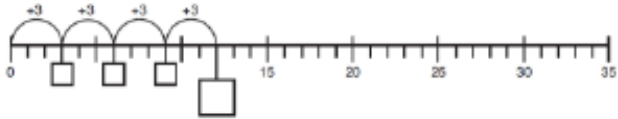

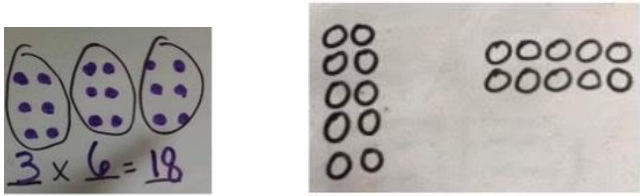
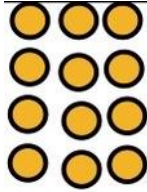

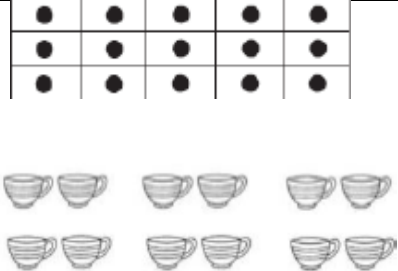
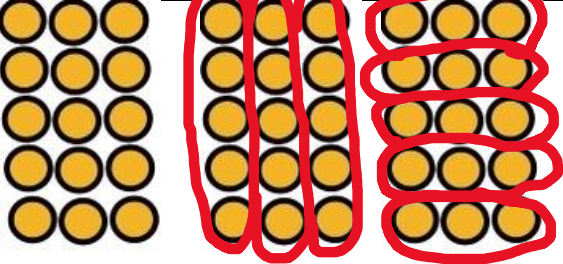
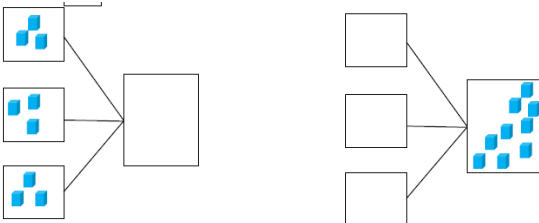
0	10	20	30	40					
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Write in sequences with multiples of numbers.

$$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = \square$$

$$3 \times 4 = \square$$

**Repeated addition**

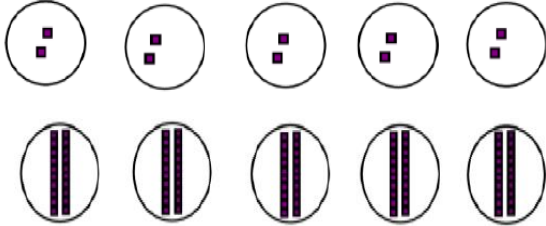
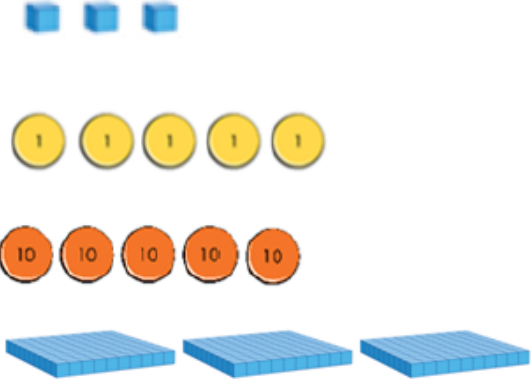
<p>Skip Counting in multiples of 2, 3, 5, and 10 from zero.</p>			
<p>Multiplication is commutative</p>	<p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of multiplication does not affect the answer.</p> 	<p>Children to represent the arrays pictorially. Make sure the arrays are drawn in different orientations to find the commutativity.</p> 	<p>Children to be able to use an array to write multiplication sentences and calculations</p>  <p> <math>3 + 3 + 3 + 3 = 12</math>  <math>4 + 4 + 4 = 12</math>  <math>4 \times 3 = 12</math>  <math>3 \times 4 = 12</math> </p>
<p>Arrays represent multiplication equations.</p>			<p><math>3 \times 5 =</math></p>
<p>Using the inverse</p> <p>This should be taught alongside division, so pupils learn how they work</p>	 <p>Children to use equipment</p>	<p>Use the part, part, whole model to show inverse relationship between multiplication and division.</p> 	<p><math>3 \times 3 =</math> <input type="text"/></p> <p><math>9 \div 3 =</math> <input type="text"/></p>

alongside each other.	to build arrays	children split the array into equal groups.	$15 \div 3 = 5$ $15 \div 5 = 3$		
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**Multiplication**

**Year 3**

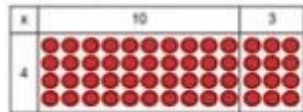
Doubling to derive new multiplication facts			$3 \times 3 = 9$ <hr/> $3 \times 6 = \text{double } 9 = 18$
Skip counting 2,3,4,5,6,8,10 from 0.			$3 \times 1 =$ $3 \times 2 =$ $3 \times 3 =$ $3 \times 4 =$ $3 \times 5 =$ $3 \times 6 =$ $3 \times 7 =$ $3 \times 8 =$
Use part-part whole model and bar model to establish the commutative relationship between multiplication and division.			$5 \times 3 = 15$ $3 \times 5 = 15$ $15 \div 3 = 5$ $15 \div 5 = 3$

<p>Ten Times greater</p>	<p>Pupil's work on this must be firmly based on concrete representations – the language of ten times greater must be well modelled and understood to prevent the numerical misconception of 'adding a zero'.</p>		<p><math>2 \times 10 = 20</math></p> <p><math>10 \times 10 = 100</math></p>
<p>Multiplying by ten and one hundred</p>			<p><math>3 \times 1 = 3</math></p> <p><math>5 \times 1 = 5</math></p> <p><math>5 \times 10 = 50</math></p> <p><math>3 \times 100 = 300</math></p>

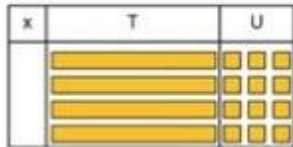
Multiplication of 2-digit numbers with partitioning (no regrouping)

Show the link with arrays to first introduce the grid method.

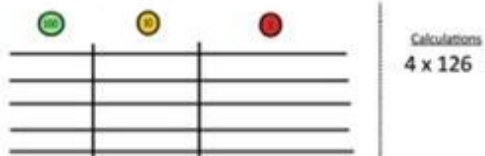
4 rows of 10  
4 rows of 3



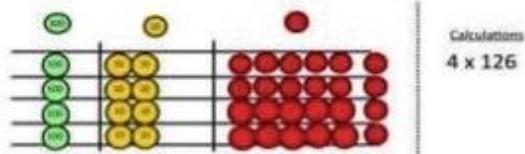
Move on to using Base 10 to move towards a more compact method.



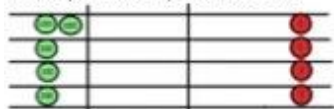
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



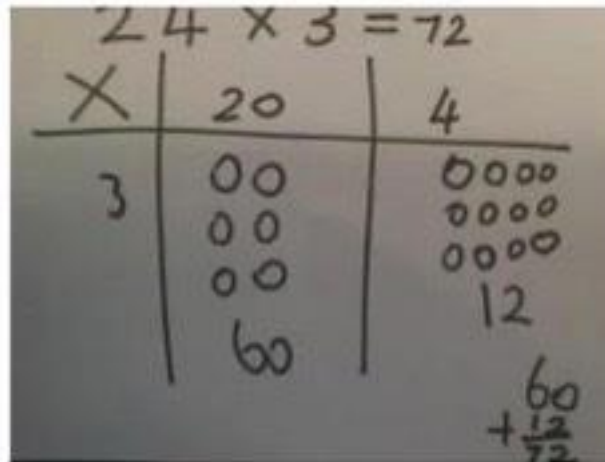
Fill each row with 126



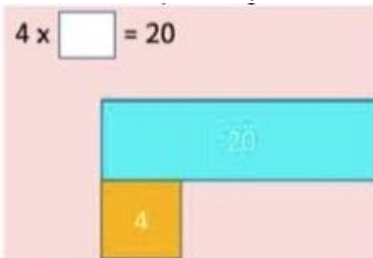
Add up each column starting with the ones making any exchanges needed. Then you have your answer.



Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking.



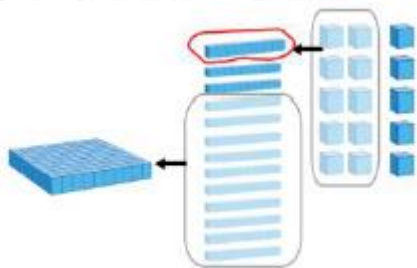
Use bar models to explore missing numbers.



x	10	2
3	≡	⋮

x	10	2
3	30	6

Multiplication of 2-digit numbers with partitioning (regrouping)



x	10	4
3	≡	⋮
	30	12

x	10	4
3	30	12

	<p><math>14 \times 3 = 42</math></p>		
--	--------------------------------------	--	--

**Multiplication**

**Year 4**

<p>Multiplying by 10 and 100</p>	<table border="1"> <thead> <tr> <th>thousands</th> <th>hundreds</th> <th>tens</th> <th>ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td>3</td> <td>3</td> </tr> <tr> <td>3</td> <td>0</td> <td>0</td> <td>0</td> </tr> </tbody> </table>	thousands	hundreds	tens	ones			3	3	3	0	0	0		<p><math>3 \times 10 = 30</math></p> <p><math>3 \times 100 = 300</math></p> <p><math>3 \times 1000 = 3000</math></p>
thousands	hundreds	tens	ones												
		3	3												
3	0	0	0												
<p>Using known facts and place value for mental multiplication involving multiples of 10 and 100</p>	<p>factor    factor    product</p> <p><math>3 \times 7 = 21</math></p> <p>factor    factor    product</p> <p><math>7 \times 3 = 21</math></p>		<p><math>30 \times 7 = 210</math>      <math>300 \times 7 = 2100</math></p> <p><math>70 \times 3 = 210</math>      <math>700 \times 3 = 2100</math></p> <p><math>7 \times 30 = 210</math>      <math>7 \times 300 = 2100</math></p> <p><math>3 \times 70 = 210</math>      <math>3 \times 700 = 2100</math></p>												
<p>Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using scaling down</p>	<table border="1"> <thead> <tr> <th>hundreds</th> <th>tens</th> <th>ones</th> </tr> </thead> <tbody> <tr> <td></td> <td>3</td> <td>3</td> </tr> <tr> <td></td> <td>3</td> <td>0</td> </tr> <tr> <td></td> <td></td> <td>3</td> </tr> </tbody> </table>	hundreds	tens	ones		3	3		3	0			3		<p><b><math>3 \times 10 = 30</math></b></p> <p><b><math>30 \div 10 = 3</math></b></p>
hundreds	tens	ones													
	3	3													
	3	0													
		3													

<p>Multiplying by partitioning one number and multiplying each part</p>			$30 \times 6 + 4 \times 6$ $14 \times 6$
<p>Mental multiplication of three 1-digit numbers, using the associative law</p>			$(4 \times 2) \times 7 \text{ or } 4 \times (2 \times 7)$ <p><b>Distributive law</b></p> <p>Four pots each containing two flowers which each have seven petals. How many petals in total?</p>
<p>Short multiplication of 3-digit number by 1-digit number</p>			$\begin{array}{r} 241 \\ \times \quad 3 \\ \hline 723 \\ \hline 1 \end{array}$

Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc.  
 When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater.

Thousands	Hundreds	Tens	Ones	•	tenths	hundredths
	100 100 100		1 1	•	0.01	0.1 0.1 0.1 0.1
1000 1000 1000		10 10	1	•	0.01 0.01 0.01 0.01	

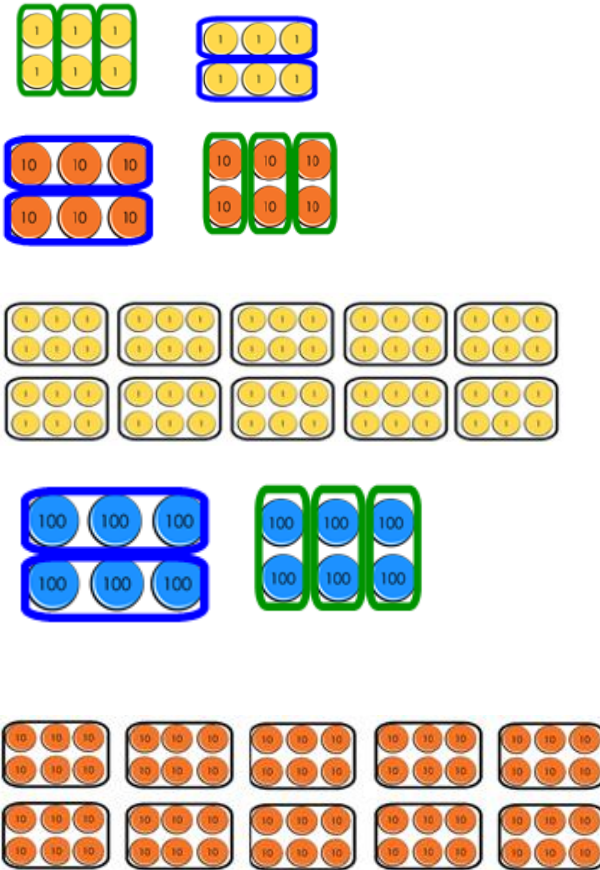
When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.  
 When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller.  
 E.g.  $210 \div 10 = 21$

Hundreds	Tens	Ones	•	tenths	hundredths
100 100	10		•	0.1 0.1 0.1 0.1	
	10 10	1	•		0.01 0.01 0.01 0.01

$$102.14 \times 10 = 1021.4$$

$$210.3 \div 10 = 21.03$$

Using known facts and place value to derive multiplication fact



$$2 \times 3 = 6$$

$$3 \times 2 = 6$$

$$2 \times 30 = 60 \quad 3 \times 20 = 60$$

$$30 \times 2 = 60 \quad 20 \times 3 = 60$$

$$2 \times 3 \times 10 = 60$$

$$2 \times 300 = 600$$

$$300 \times 2 = 600$$

$$3 \times 200 = 600$$

$$200 \times 3 = 600$$



$$2 \times 30 \times 10 = 600$$

$$20 \times 3 \times 10 = 600$$

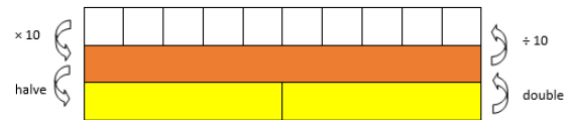
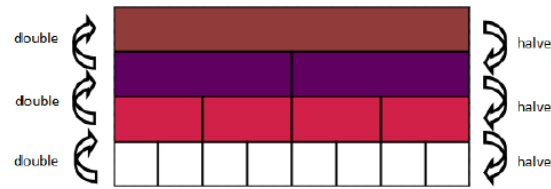
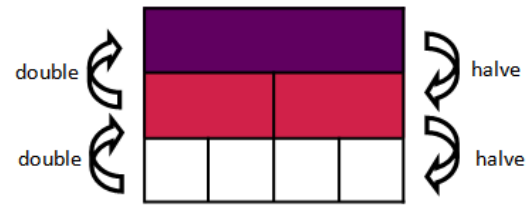
$$20 \times 30 = 600$$

$$30 \times 20 = 600$$

These are the multiplication facts pupils should be able to derive from a known fact.

2 100 000	700 000 x 3	70 000 x 30	7000 x 300	700 x 3000	70 x 30 000	7 x 300 000
210 000	70 000 x 3	7000 x 30	700 x 300	70 x 3000	7 x 30 000	
21 000	7000 x 3	700 x 30	70 x 300	7 x 3000		
2100	700 x 3	70 x 30	7 x 300			
210	70 x 3	7 x 30				
<b>21</b>	<b>= 7 x 3</b>					
2.1	0.7 x 3	7 x 0.3				
0.21	0.07 x 3	0.7 x 0.3	7 x 0.03			
0.021	0.007 x 3	0.07 x 0.3	0.7 x 0.03	7 x 0.003		

Doubling and halving



Multiply by 4 by doubling and doubling again  
e.g.  $16 \times 4 = 32 \times 2 = 64$

Divide by 4 by halving and halving again  
e.g.  $104 \div 4 = 52 \div 2 = 26$

Multiply by 8 by doubling three times  
e.g.  $12 \times 8 = 24 \times 4 = 48 \times 2 = 96$

Divide by 8 by halving three times  
e.g.  $104 \div 8 = 52 \div 4 = 26 \div 2 = 13$

Multiply by 5 by multiplying by 10 then halving,  
e.g.  $18 \times 5 = 180 \div 2 = 90$ .

Divide by 5 by dividing by 10 and doubling,  
e.g.  $460 \div 5 = \text{double } 46 = 92$

Multiply by partitioning one number and multiplying each part

Distributive law

$a \times (b + c) = a \times b + a \times c$

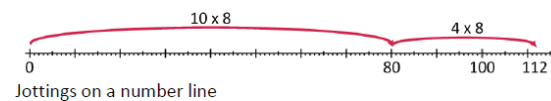


Cuisenaire rods to build arrays

Bead string where each bead has a value of 8:



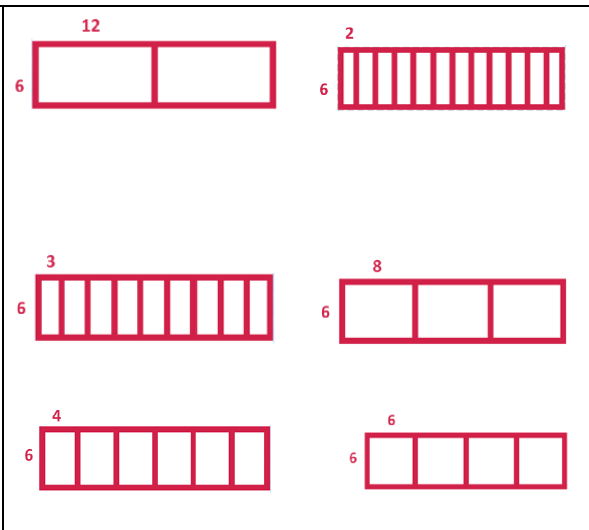
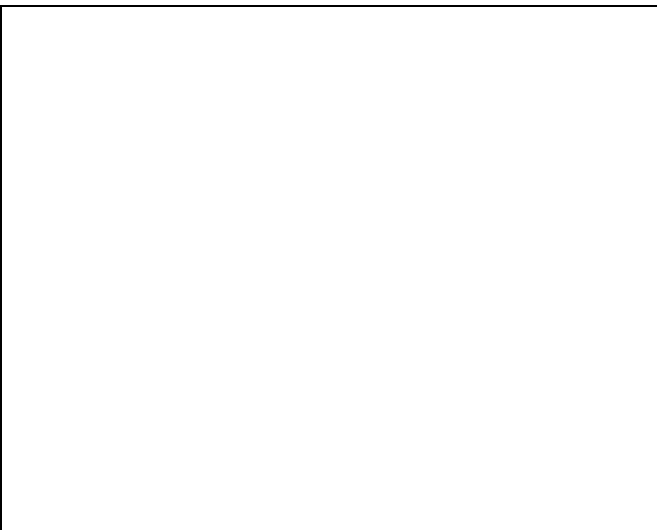
Represent with area model



Jottings on a number line

$8 \times 14 = 8 \times 10 + 8 \times 4$

Using knowledge of factors



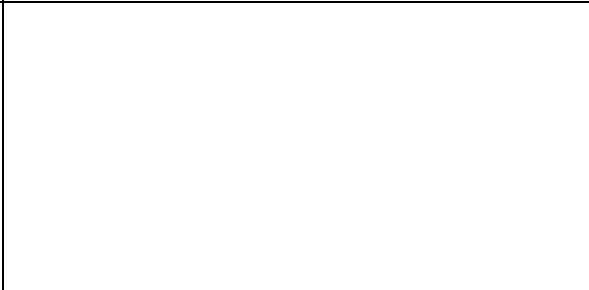
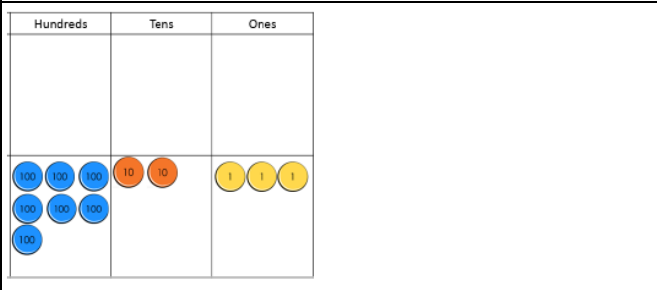
Calculate  $6 \times 24$  by using factor pairs of 24

Two and twelve are factors of 24:  
 $6 \times 2 \times 12$   
 $6 \times 12 \times 2$

Three and eight are factors of 24:  
 $6 \times 3 \times 8$   
 $6 \times 8 \times 3$

Four and six are factors of 24:  
 $6 \times 4 \times 6$   
 $6 \times 6 \times 4$

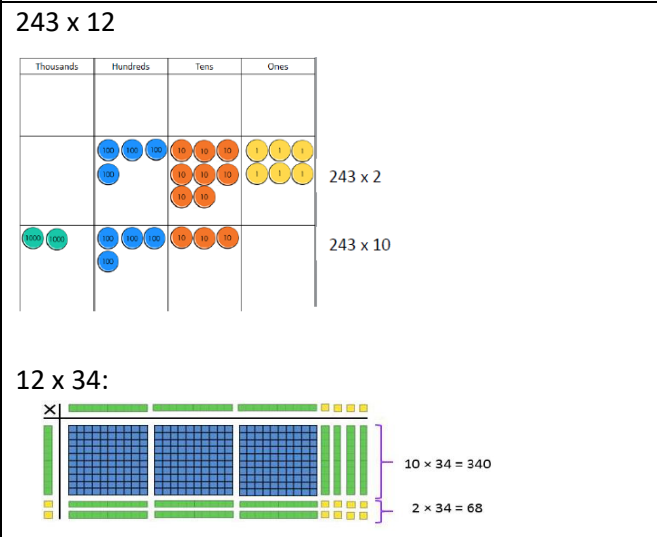
Formal written method of short multiplication



$$\begin{array}{r} 241 \\ \times \quad 3 \\ \hline 723 \\ \hline 1 \end{array}$$

Multiplying by a 2-digit number

Formal written method of long multiplication



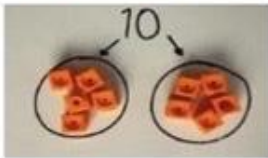
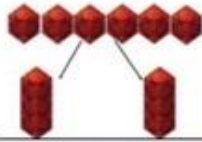
$$\begin{array}{r} 243 \\ \times \quad 12 \\ \hline 486 \\ 2430 \\ \hline \hline 34 \\ \times 12 \\ \hline 68 \\ 340 \\ \hline 408 \end{array}$$

Division

Year 1

Division as sharing

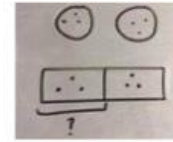
Use a range of equipment to introduce sharing.



I have 10 cubes, can you share them equally in 2 groups?

Sharing objects pictorially

12 children get into teams of 4 to play a game. How many teams are there?



6 sweets are shared between 2 people. How many do they each have?

$8 \div 2 = \square$



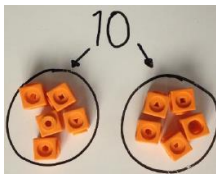
Share 9 buns between three people.  $9 \div 3 = 3$

Division

Year 2

Division as sharing

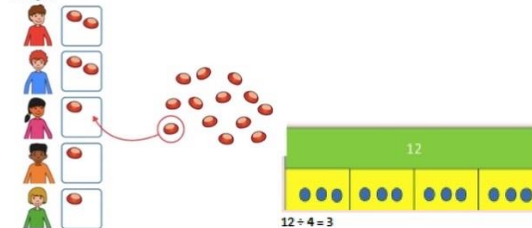
Use a range of equipment to develop sharing.



Children use pictures or shapes to share quantities.



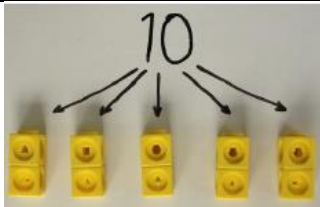
I have twenty conkers, and I share them equally between five children. How many conkers does each child get?



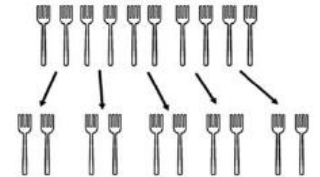
$10 \div 2 = 5$

Use bar models to support understanding.

# Division as grouping



Divide quantities into equal groups. Use cubes, counters, objects and place value counters to help understanding.



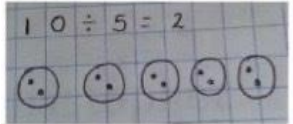
8			
2	2	2	2

Link images to bar modelling. The bar model to be the whole.

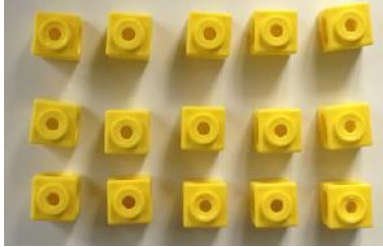
Also link number lines for grouping.

$28 \div 7 = 4$   
Divide 28 into 7 groups. How many are in each group?

Max is filling party bags with sweets. He has 10 sweets altogether and decides to put 2 in every bag. How many bags can he fill?



# Arrays and part, part, whole models to emphasise the relationship between multiplication and division.



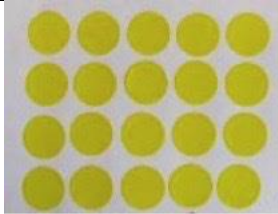
$15 \div 5 = \boxed{3}$   
 $15 \div 3 = \boxed{5}$

# Division

# Year 3

# Division as grouping

How many groups of 8 in 32?  
 $32 \div 8 = 4$



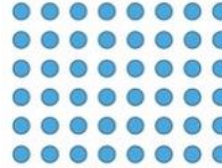
Link division to multiplication with arrays and think about the number sentences that can be created.

$$20 \div 4 = 5$$

$$20 \div 5 = 4 \quad 5 \times 4 = 20 \quad 4 \times 5$$

Draw an area and use lines to split them up into groups. Make division and multiplication sentences from them.

Array problem:



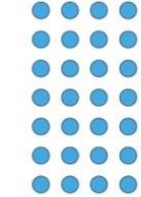
\* How many groups of six are there?

$$\square \times 6 = \square$$

\* How many groups of three are there?

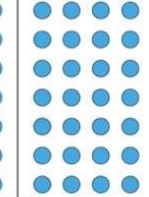
$$\square \times 3 = \square$$

'Circle the groups of four and complete the sentence.'



There are \_\_\_ groups of four.

'Circle the four equal groups and complete the sentence.'



There are four groups of \_\_\_.

Find the inverse of multiplication and division sentences.

'Fill in the missing numbers.'

$$3 \times 4 = \square$$

$$5 \times 4 = \square$$

$$4 \times 3 = \square$$

$$\square \times 4 = 20$$

$$12 \div 4 = \square$$

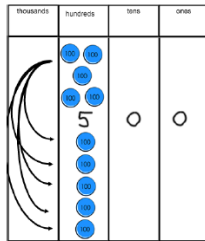
$$20 \div 4 = \square$$

'What multiplication fact can be used to solve this division calculation?'

$$24 \div 4 = ?$$

I can use this multiplication fact:  $\square \times \square = \square$

Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using grouping

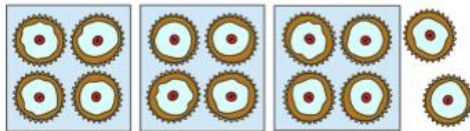


$$500 \div 100 = \square$$

My whole is 500 and the value of the equal parts is 100. How many parts are there?

Division with remainders

'A baker has fourteen cakes. He sells cakes in boxes of four. How can he box the cakes?'

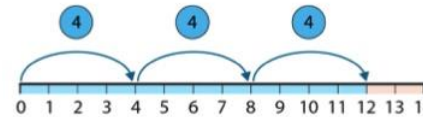


$$14 \div 4 = 4 \text{ r } 2$$



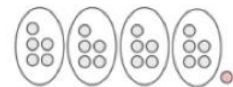
Use objects between groups and see how much is left over.

'A baker has fourteen cakes. He sells cakes in boxes of four. How can he box the cakes?'



- 'One box of four is four.'
- 'Two boxes of four are eight.'
- 'Three boxes of four are twelve.'
- 'There are two cakes left over.'

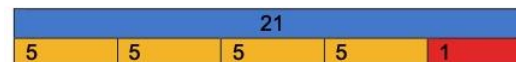
Use a number line to jump equally. Then you will see how many more you need to jump to find the remainder.



$$21 \div 5 = 4 \text{ r } 1$$

Draw dots and group them to divide an amount and clearly show a remainder.

Use bar models to show division with remainders



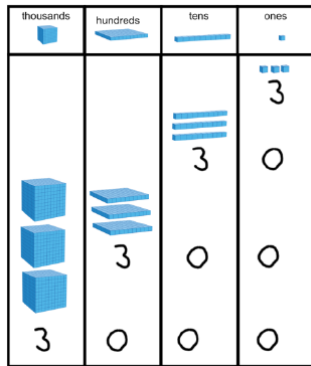
Complete written divisions and show remainders using r.

14	÷	4	=	3	r	2
dividend	÷	divisor	=	quotient	r	remainder

$$22 \div 4 = \square \text{ r } \square$$

$$23 \div 4 = \square \text{ r } \square$$

Dividing by 10 and 100



$$30 \div 10 = 3$$

$$300 \div 100 = 3$$

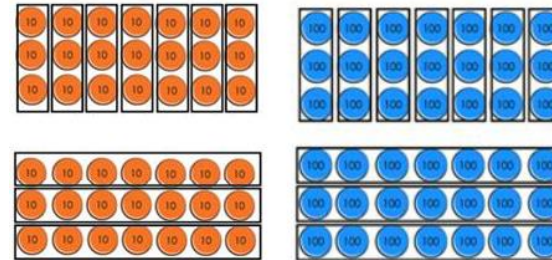
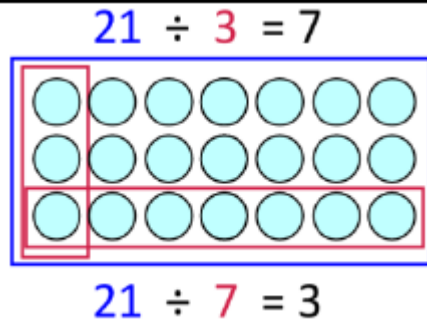
$$3000 \div 1000 = 3$$

$$300 \div 10 = 30$$

$$3000 \div 100 = 30$$

$$3000 \div 10 = 300$$

Derived facts



$$210 \div 7 = 30$$

$$210 \div 3 = 70$$

$$210 \div 30 = 7$$

$$210 \div 70 = 3$$

$$2100 \div 7 = 300$$

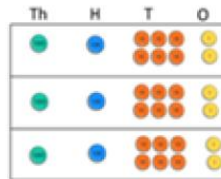
$$2100 \div 3 = 700$$

$$2100 \div 300 = 7$$

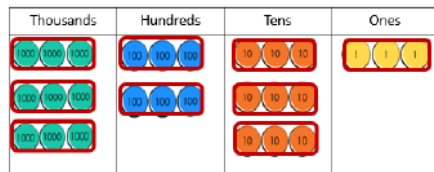
$$2100 \div 700 = 3$$

Short division of 4-digit numbers by 1-digit numbers

Division as sharing



Division as grouping



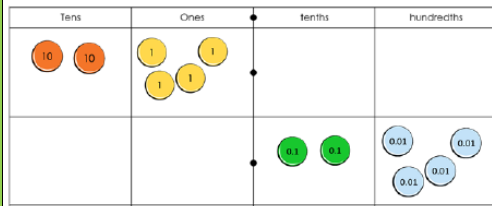
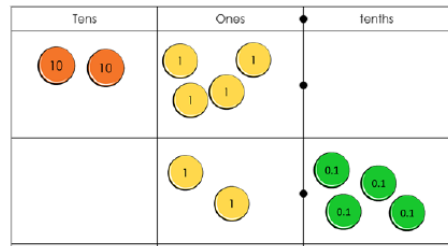
$$3 \overline{) 3486}$$

Division as sharing

Division as grouping

$$\begin{array}{r} 3231 \\ 3 \overline{) 9693} \end{array}$$

Division of a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths



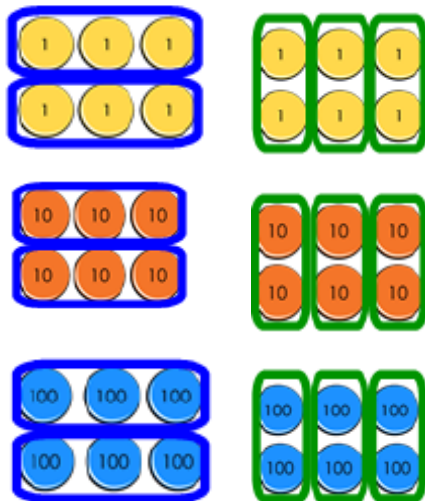
$$24 \div 10 = 2.4$$

$$24 \div 100 = 0.24$$

## Division

Year 5 & 6

Deriving facts from known facts



$$6 \div 2 = 3$$

$$6 \div 3 = 2$$

$$60 \div 2 = 30$$

$$60 \div 30 = 2$$

$$60 \div 3 = 20$$

$$60 \div 20 = 3$$

$$600 \div 2 = 300$$

$$600 \div 300 = 2$$

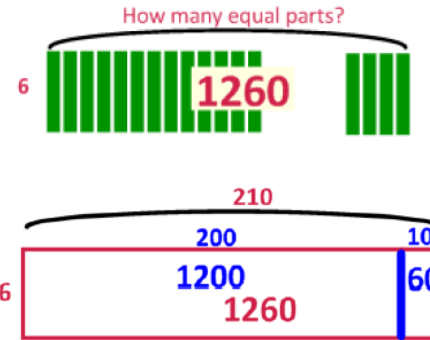
$$600 \div 3 = 200$$

$$600 \div 200 = 3$$

Using knowledge of multiples to divide



$$112 \div 8 = 80 \div 8 + 32 \div 8$$



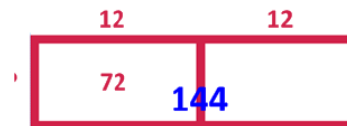
$$1260 \div 6 = 1200 \div 6 + 60 \div 6$$

Using knowledge of factors to divide



I know 2 and 12 are a factor pair of 24 and so I can divide by 2 and then by 12.

$$144 \div 24$$



$$144 \div 2 \div 12$$

Short division

Dividing a 4-digit numbers by 1-digit numbers

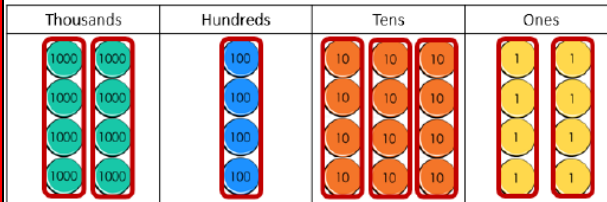


$$8528 \div 4$$

$$\begin{array}{r} 2132 \\ 4 \overline{) 8528} \end{array}$$

8 thousand shared into 4 equal groups  
 5 hundred shared into 4 equal groups  
 Regroup 1 hundred for 10 tens  
 12 tens shared into 4 equal groups  
 8 ones shared into 4 equal groups.

Grouping

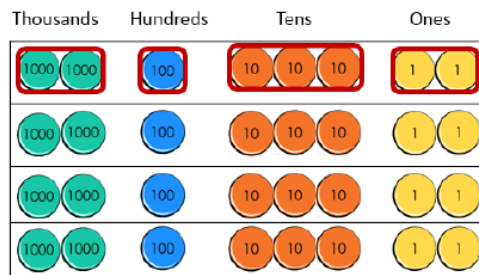


How many groups of 4 thousand in 8 thousand?  
 How many groups of 4 hundred in 5 hundred?  
 Regroup 1 hundred for 10 tens.  
 How many groups of 4 tens in 12 tens?  
 How many groups of 4 ones in 8 ones?

Short division

Dividing a 4-digit numbers by 1-digit numbers

Sharing

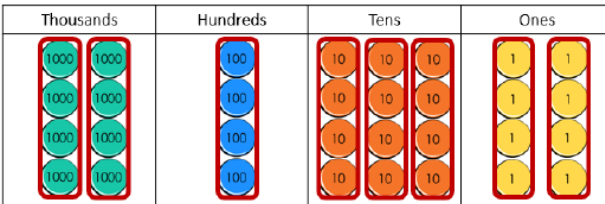


8 thousand shared into 4 equal groups  
 5 hundred shared into 4 equal groups  
 Regroup 1 hundred for 10 tens  
 12 tens shared into 4 equal groups  
 8 ones shared into 4 equal groups.

$$8528 \div 4$$

$$\begin{array}{r} 2132 \\ 4 \overline{) 8528} \end{array}$$

### Grouping

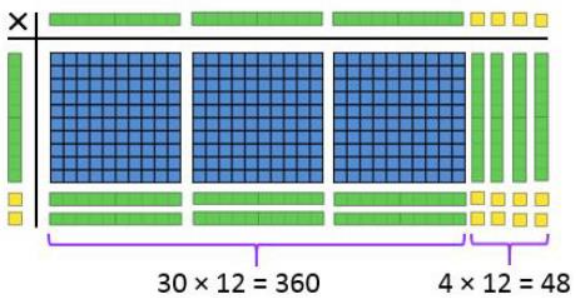


How many groups of 4 thousand in 8 thousand?  
 How many groups of 4 hundred in 5 hundred?  
 Regroup 1 hundred for 10 tens.  
 How many groups of 4 tens in 12 tens?  
 How many groups of 4 ones in 8 ones?

### Long division

Dividing a 4-digit number by a 2-digit number

$$408 \div 12$$



$$\begin{array}{r}
 34 \\
 12 \overline{) 408} \\
 \underline{36} \phantom{0} \\
 48 \\
 \underline{48} \\
 0
 \end{array}$$

Long division (remainder)

Start with a remainder in the ones.

$$\begin{array}{r} 146 \\ 35 \overline{) 5110} \\ \underline{16} \phantom{0} \\ 21 \phantom{0} \\ \underline{21} \\ 0 \end{array}$$

$$\begin{array}{r} \text{h t o} \\ 041R1 \\ 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).  
 4 goes into 16 four times.  
 4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400R7 \\ 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).  
 8 goes into 32 four times (3,200 ÷ 8 = 400)  
 8 goes into 0 zero times (tens)  
 8 goes into 7 zero times, and leaves a remainder of 7.

$$\begin{array}{r} \text{h t o} \\ 061 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply  $1 \times 4 = 4$ , write that four under the 7, and subtract. This finds us the remainder of 3.

Check:  $4 \times 61 + 3 = 247$

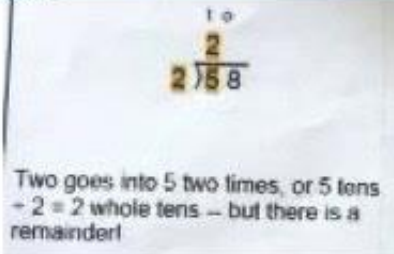
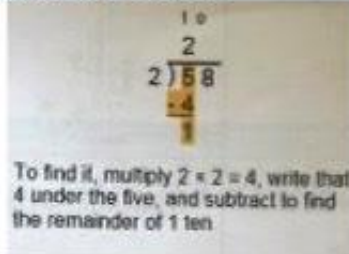
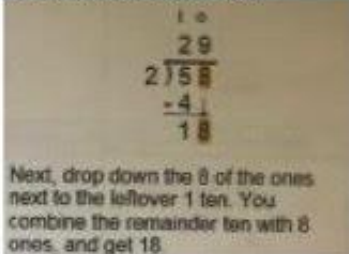
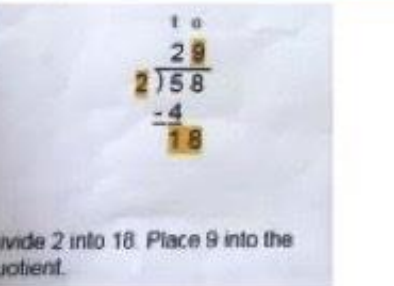
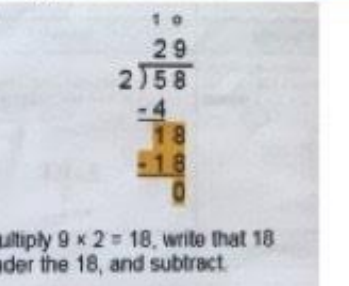
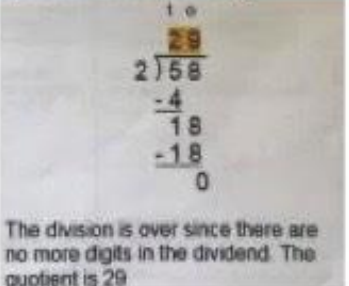
$$\begin{array}{r} \text{th h t o} \\ 0402 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply  $2 \times 4 = 8$ , write that eight under the 9, and subtract. This finds us the remainder of 1.

Check:  $4 \times 402 + 1 = 1,609$



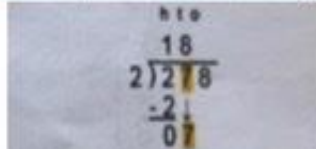
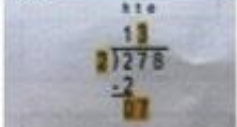
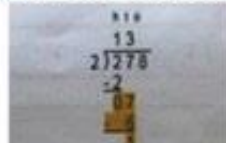



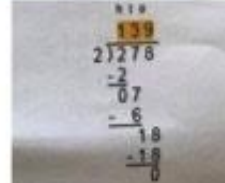
Long division (remainder)

A remainder in the tens.

1. Divide	2. Multiply and Subtract	3. Drop down the next digit.
 <p>Two goes into 5 two times, or 5 tens - 2 = 2 whole tens - but there is a remainder!</p>	 <p>To find it, multiply <math>2 \times 2 = 4</math>, write that 4 under the five, and subtract to find the remainder of 1 ten</p>	 <p>Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18</p>
 <p>Divide 2 into 18. Place 9 into the quotient.</p>	 <p>Multiply <math>9 \times 2 = 18</math>, write that 18 under the 18, and subtract.</p>	 <p>The division is over since there are no more digits in the dividend. The quotient is 29</p>

Long division (remainder).

A remainder in any of the place values.

1. Divide	2. Multiply and Subtract	3. Drop down the next digit.
 <p>Two goes into 2 one time, or 2 hundreds <math>\div 2 = 1</math> hundred.</p>	 <p>Multiply <math>1 \times 2 = 2</math>, write that 2 under the two, and subtract to find the remainder of zero.</p>	 <p>Next, drop down the 7 of the tens next to the zero.</p>
4. Divide	5. Multiply and Subtract	6. Drop down the next digit.
 <p>Divide 2 into 7. Place 3 into the quotient.</p>	 <p>Multiply <math>3 \times 2 = 6</math>, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	 <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
7. Divide	8. Multiply and Subtract	9. Drop down the next digit.
 <p>Divide 2 into 18. Place 9 into the quotient.</p>	 <p>Multiply <math>9 \times 2 = 18</math>, write that 18 under the 18, and subtract to find the remainder of zero.</p>	 <p>There are no more digits to drop down. The quotient is 139.</p>

